Physics 2014
Midterm 2

Spring 2005
March 10th (Thursday)
5:30 - 6:30 pm

Name (PRINT):

Name (signed):

Recitation Section/Time:

Name of your Recitation TA:

Instructions

This is a closed book exam of one hour duration. You are allowed to use a calculator without any pre-programmed information. There are four problems, with unequal weights. Allocate your time judiciously. Make sure that you use proper units and 3 significant figures accuracy, unless specified otherwise. Problem 1 contains five parts in multiple choice format. Each bit of problem 1 carries 3 points, for a total of 15 points. Circle your answer among the choices given. Problem 2 has five short parts, each carrying 7 points. Write your answer in the space provided, indicating your logic and the relevant intermediate steps. Problems 3 and 4 each carries 25 points. These problems require you to show your work. Any partial credit you earn for Problems 2, 3 and 4 will depend on the intermediate steps you show and on its clarity. If the space provided is insufficient, use the back of the page, indicating the appropriate problem/bit number.

Good Luck
Problem 1: 15 points (3 points per bit)

Circle the correct answer for each of the following five questions.

(i) An object is moving to the right, and is experiencing a net force that is directed to the right. The magnitude of the force is decreasing with time. The speed of the object is

(A) decreasing
(B) increasing
(C) constant in time
(D) can either be increasing or decreasing

(ii) A woman is straining to lift a large crate, without success. It is too heavy. We denote the force on the crate as follows. \( P \) is the upward force exerted on the crate by the person, \( C \) is the contact force on the crate by the floor, and \( W \) is the weight of the crate. How are the magnitudes of these forces related, while the person is trying unsuccessfully to lift the crate?

(A) \( P + C < W \)
(B) \( P + C > W \)
(C) \( P = C \)
(D) \( P + C = W \)

(iii) Joe and Bill are playing tug-of-war. Joe is pulling with a force of 200 N, Bill is simply hanging on to the rope. Neither person is moving. What is the tension in the rope?

(A) 0 N
(B) 200 N
(C) 400 N
(D) 300 N

(iv) Bob and Ann are standing on top of a tower. Bob drops a 0.3 kg stone from the tower vertically downward with zero initial velocity. At the same instant in time, Ann throws a 0.5 kg stone in the horizontal direction. The ground below the tower is flat. Which statement correctly describes the subsequent motion of the stones?

(A) The 0.3 kg stone hits the ground first
(B) The 0.5 kg stone hits the ground first
(C) Both stones hit the ground at the same time
(D) Insufficient data to determine which stone reaches ground first
(v) You are in an elevator that has descended from the 50th floor and coming to a halt at the first floor. As it does, your apparent weight is

(A) More than your true weight
(B) Less than your true weight
(C) Equal to your true weight
(D) Zero

Problem 2: 35 points (7 points for each of the five bits)

This problem has five short parts, each carrying 7 points. Make sure that you include all intermediate steps necessary.

(i) The position of a 2.00 kg object as a function of time is given by \( \vec{r} = (2.1) t^3 \hat{i} + (1.7) t \hat{j} \), where \( t \) is measured in seconds and \( r \) is in meters. What is the force on the object (magnitude and direction) at \( t = 4.7 \) s?

\[
\vec{v} = 3(2.1) t^2 \hat{i} + 1.7 \hat{j},
\]

\[
\vec{a} = (2)(3)(2.1) \frac{m}{s^2} \hat{i} = 12.6 \hat{i}.
\]

\[
= (12.6)(4.7) \frac{m}{s^2} \hat{i} = (59.2 \frac{m}{s^2}) \hat{i}.
\]

\[
\vec{F} = m\vec{a} = 2 \times 59.2 \ N \hat{i} = 118.4 \ N \hat{i}.
\]
(ii) A child on a sled starts from rest at the top of a $15.0^\circ$ slope. The trip to the bottom takes 22.6 s. How long is the slope? Ignore forces of friction. Show a diagram and indicate your logic.

\[
\begin{align*}
\alpha_x &= g \sin \theta = (9.8 \text{ m/s}^2) \sin 15^\circ \\
&= 2.54 \text{ m/s}^2
\end{align*}
\]

\[
\begin{align*}
\chi_f - \chi_i &= \alpha_{x_i} t + \frac{1}{2} \alpha_x t^2 \\
&= 0 + \frac{1}{2} (2.54) (22.6)^2 \text{ m} \\
&= 648.7 \text{ m}
\end{align*}
\]

(iii) A 21.0 kg box must be slid across a horizontal floor. If the coefficient of static friction between the box and the floor is 0.37, what is the minimum force needed to start the box moving from rest? Show a free body diagram and indicate your logic.

\[
\begin{align*}
\mathbf{F} &= \mathbf{f_s} + \mathbf{n} \\
W &= n \\
\mathbf{F} > \mathbf{f_s, max} \Rightarrow \mathbf{F} > \mu_s n \Rightarrow \\
\mathbf{F} > \mu_s mg = (0.37)(21.0)(9.8) N = 76.1 N \\
\mathbf{F} > 76.1 \text{ N}
\end{align*}
\]
(iv) A 1.25 kg block is sitting on top of an inclined plane with an inclination angle of 35.0° from the horizontal. A force is applied parallel to the inclined plane in order to push the block down the incline. The minimum applied force needed to move the block is 2.50 N. Determine the coefficient of static friction between the block and the incline. Make sure that you include a free body diagram and explain your reasoning.

\[
\begin{align*}
N - mg \cos \theta &= 0 \\
F + mg \sin \theta - f_s &> 0 \\
F > f_s + mg \sin \theta \\
F &= mg (\mu_s \cos \theta - \sin \theta) \\
\Rightarrow \mu_s &= 0.949
\end{align*}
\]

\[
\begin{align*}
2.50 &= \mu_s (1.25) (9.80 \cos 35° - 1.25 \sin 35°) \\
&= (1.25) (9.80) \sin 35°
\end{align*}
\]

(v) A cat leaps to catch a bird. If the cat’s jump was at 60.0° off the ground and its initial velocity was 7.22 m/s, what is the highest point of its trajectory?

\[
\begin{align*}
\mathbf{v}_x &= u \cos \theta = 7.22 \cos 60° = 3.61 \text{ m/s} \\
\mathbf{v}_y &= u \sin \theta = 7.22 \sin 60° = 6.25 \text{ m/s} \\
\mathbf{v}_y &= \mathbf{v}_y - g t \\
0 &= 6.25 - 9.8 t \Rightarrow t = \frac{6.25}{9.8} = 0.64 \text{ s} \\
\mathbf{v}_f &= \mathbf{v}_i + \frac{g t^2}{2} = \mathbf{v}_i - g t \\
\mathbf{v}_y &= (6.25)(0.64) - \frac{1}{2}(9.80)(0.64)^2 \\
\Delta y &= 2.00 \text{ m/s}
\end{align*}
\]
Problem 3: 25 points

A 2.50 kg block is launched up a steel ramp that is inclined at 25.0° from the horizontal. The block's initial speed is 15.0 m/s. The coefficient of kinetic friction between the block and the steel ramp is 0.30.

(a) Draw a free body diagram indicating all the forces acting on the block. [5 pts]

(b) Determine the maximum distance the block would travel up on the ramp from the starting point. [10 pts]

\[ n - mg \cos \theta = 0 \]
\[ -f - mg \sin \theta = ma_x \Rightarrow \]
\[ -f - mg \cos \theta - mg \sin \theta = ma_x \quad \text{or} \]
\[ a_x = -g \left( \mu_k \cos \theta + \sin \theta \right) = -9.80 \left( 0.30 \cos 25^\circ + \sin 25^\circ \right) \]
\[ = -6.81 \text{ m/s}^2 \]
\[ v_x^2 = v_{x_0}^2 + 2a_x \Delta x \Rightarrow 0 = (15)^2 - 2(6.81)(\Delta x) \]
\[ \Delta x = \frac{(15)^2}{2(6.81)} = 16.52 \text{ m} \]
(c) When the block reaches its maximum height on the incline, it stops momentarily and slides down the incline. What would be the block's velocity when it returns to the starting point where it was launched?

$$\sum F = ma$$

$$m g \sin 8 - \mu_k m g \cos 8 = m a_x$$

$$a_x = g (\mu_k \cos 8 - \sin 8) = -9.80 \left(0.70 \cos 25^\circ - \sin 25^\circ\right) = -1.48 \text{ m/s}^2$$

$$\Delta x_f = \sqrt{\Delta x_i^2 + 2 \Delta x a_x} \Rightarrow$$

$$v_{xf} = 6.99 \text{ m/s}$$

(d) If the coefficient of kinetic friction between the block and the same incline were 0.80 (instead of 0.30), describe how its motion would be affected, giving your reasoning.

[5 pts]

For motion up incline, 
$$a_x = -9.80 \left(0.80 \cos 25^\circ - \sin 25^\circ\right) = -11.25 \text{ m/s}^2$$

$$\Delta x = \frac{15^2}{2 \left(11.25\right)} = 10.0 \text{ m}$$

For downward motion, if allowed, 
$$a_x = -9.80 \left(0.80 \cos 25^\circ - \sin 25^\circ\right) = -2.96 \text{ m/s}^2$$

$$\Rightarrow$$ Block will not slide down.
Problem 4: 25 points

A firefighter 50.0 m away from a burning building directs a stream of water from a fire hose that he holds at a height of 0.80 m above ground at an angle of 30.0° above the horizontal.

(a) If the speed of the stream is 40.0 m/s, at what height will the water strike the building? Give your reasoning along with a sketch of the situation. [18 pts]

\[ \begin{align*}
    d &= \sqrt{2} \cdot 50 \text{ m} \\
    y_f &= y_0 + v_y \sin \theta - \frac{1}{2} g \frac{d^2}{y_0 + v_y \sin \theta} \\
    y_f &= \frac{1}{2} g \frac{d^2}{v_y \sin \theta} \\
    y_f &= 0.80 + 50 \tan 30^\circ - \frac{1}{2} \left( \frac{50}{(40 \cos 30^\circ)} \right)^2 \\
    y_f &= 0.80 + 20 \frac{\sqrt{3}}{2} - 10 \cdot 0.21 \\
    y_f &= 19.46 \text{ m}
\end{align*} \]

(b) If the water stream is to hit the building at a height of 10.5 m, at what angle(s) above the horizontal should the firefighter aim the hose with the speed of the stream the same as 40.0 m/s? [7 pts]

Useful hint: The solutions to the equation \( A \cos^2 \theta + B \sin \theta \cos \theta + C = 0 \) are

\[
\cos^2 \theta = \frac{B^2 - 2AC \pm \sqrt{B^2 - 4AC - 4C^2}}{2(A^2 + B^2)}
\]

\[\begin{align*}
    y_f &= 10.5 \text{ m} \\
    10.5 &= 0.80 + 50 \tan \theta - \frac{1}{2} \left( \frac{50}{(40 \cos \theta)} \right)^2 \\
    9.7 \cos^2 \theta - 50 \sin \theta \cos \theta + 7.66 &= 0
\end{align*}\]

\[
\cos^2 \theta = \frac{(50)^2 - 2(9.7)(7.66) \pm \sqrt{(50)^2 - 4(9.7)(7.66)(7.66) - 4(7.66)^2}}{2(9.7^2)}
\]

\[
= 0.453 \pm 0.428 = 20.2^\circ \text{ or } 79.8^\circ
\]

\( \text{Need to keep } 2 \text{ digits for accuracy.} \)