6.2. **Model:** The boat is treated as a particle whose motion is governed by constant-acceleration kinematic equations in a plane.

**Visualize:**

**Pictorial representation**

**Solve:** Resolving the acceleration into its $x$ and $y$ components, we obtain

$$\vec{a} = (0.80 \text{ m/s}^2) \cos 40^\circ \hat{i} + (0.80 \text{ m/s}^2) \sin 40^\circ \hat{j} = (0.613 \text{ m/s}^2) \hat{i} + (0.514 \text{ m/s}^2) \hat{j}$$

From the velocity equation $\vec{v}_t = \vec{v}_0 + \vec{a}(t_1 - t_0)$,

$$\vec{v}_t = (5.0 \text{ m/s}) \hat{i} + \left[(0.613 \text{ m/s}^2) \hat{i} + (0.514 \text{ m/s}^2) \hat{j}\right](6 \text{ s} - 0 \text{ s}) = (8.68 \text{ m/s}) \hat{i} + (3.09 \text{ m/s}) \hat{j}$$

The magnitude and direction of $\vec{v}$ are

$$v = \sqrt{(8.68 \text{ m/s})^2 + (3.09 \text{ m/s})^2} = 9.21 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{3.09 \text{ m/s}}{8.68 \text{ m/s}}\right) = 20^\circ \text{ north of east}$$

**Assess:** An increase of speed from 5.0 m/s to 9.21 m/s is reasonable.
6.10. **Model:** The spheres will be treated as particles that move according to the constant-acceleration kinematic equations.

**Visualize:**

---

**Known**

- \( x_0A = x_0B = t_0A = t_0B = 0 \)
- \( y_0A = y_0B = 1.0 \text{ m} \)
- \( y_{1A} = y_{1B} = 0 \)
- \( m_A = 1.0 \text{ kg} \)
- \( m_B = 0.4 \text{ kg} \)
- \( (v_{0A})_y = 5.0 \text{ m/s} \)
- \( (v_{0B})_y = 2.5 \text{ m/s} \)
- \( (v_{0A})_x = (v_{0B})_x = 0 \)
- \( a_y = -g \)

**Find**

- \( t_{1A} \quad t_{1B} \)
- \( x_{1A} \quad x_{1B} \)

---

**Solve:** Using \( y_{1A} = y_{0A} + (v_{0A})_y(t_{1A} - t_{0A}) + \frac{1}{2}(a_y)(t_{1A} - t_{0A})^2 \), we get

\[
-1.0 \text{ m} = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}(-9.8 \text{ m/s}^2)(t_{1A} - 0)^2 \Rightarrow t_{1A} = 0.452 \text{ s} = t_{1B}
\]

Both take the same time to reach the floor.

We are now able to calculate \( x_{1A} \) and \( x_{1B} \) as follows:

- \( x_{1A} = x_{0A} + (v_{0A})_x(t_{1A} - t_{0A}) + \frac{1}{2}(a_x)(t_{1A} - t_{0A})^2 = 0 \text{ m} + (5.0 \text{ m/s})(0.452 \text{ s} - 0 \text{ s}) + 0 \text{ m} = 2.26 \text{ m} \)
- \( x_{1B} = x_{0B} + (v_{0B})_x(t_{1B} - t_{0B}) + \frac{1}{2}(a_x)(t_{1B} - t_{0B})^2 = 0 \text{ m} + (2.5 \text{ m/s})(0.452 \text{ s} - 0 \text{ s}) + 0 \text{ m} = 1.13 \text{ m} \)

**Assess:** Note that \( t_{1B} = t_{1A} \) since both the spheres move with the same vertical acceleration and both of them start with zero vertical velocity. The horizontal distance for sphere B is one-half the distance for sphere A because the horizontal velocity of sphere B is one-half that of A.
6.16. **Model:** The position vectors $\vec{r}$ and $\vec{r}'$ in frames $S$ and $S'$ are related by the equation $\vec{r} = \vec{r}' + \vec{R}$, where $\vec{R}$ is the position vector of the origin of frame $S'$ as measured in frame $S$. $S$ is Ted’s frame and $S'$ is Stella’s frame.

**Visualize:**

**Pictorial representation**

**Solve:** The relation between $\vec{R}$ and the velocity of Stella $\vec{V}$ is

$$\frac{\vec{R}}{S_s} = \vec{V} = (100 \text{ m/s})\cos 45^\circ \hat{i} - (100 \text{ m/s})\sin 45^\circ \hat{j}$$

$$\Rightarrow \vec{R} = (5.0 \text{ m}) \left[ \frac{100}{\sqrt{2}} \hat{i} - \frac{100}{\sqrt{2}} \hat{j} \right] = (353.6 \hat{i} - 353.6 \hat{j}) \text{ m}$$

Because $\vec{r} = \vec{r}' + \vec{R}$,

$$\vec{r}' = \vec{r} - \vec{R} = (200 \text{ m})\hat{i} - [(353.6 \text{ m})\hat{i} - (353.6 \text{ m})\hat{j}] = -(154 \text{ m})\hat{i} + (354 \text{ m})\hat{j}$$

The vector $\vec{r}'$ determines the position of the exploding firecracker as seen by Stella.

Visualize: 

Solve: (a) Using \( y_t = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2 \), 
\[
0 = 1.8 \text{ m} + v_0 \sin 40^\circ(t_1 - 0) + \frac{1}{2}(-9.8 \text{ m/s}^2)(t_1 - 0)^2 \\
= 1.8 \text{ m} + (7.713 \text{ m/s})(t_1 - (4.9 \text{ m/s}))(t_1) \Rightarrow t_1 = -0.206 \text{ s and 1.780 s}
\]
The negative value of \( t_1 \) is unphysical for the current situation. Using \( t_1 = 1.780 \text{ s} \) and \( x_t = x_0 + v_{0x}(t_1 - t_0) \), we get 
\[
x_t = 0 + (v_0 \cos 40^\circ \text{ m/s})(1.780 \text{ s} - 0 \text{ s}) = (12 \text{ m/s})\cos 40^\circ(1.78 \text{ s}) = 16.36 \text{ m}
\]

(b) We can repeat the calculation for each angle. A general result for the flight time at angle \( \theta \) is 
\[
t_1 = \frac{12\sin \theta + \sqrt{144\sin^2 \theta + 35.28}}{9.8 \text{ s}}
\]
and the distance traveled is \( x_t = 12\cos \theta \times t_1 \). We can put the results in a table.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( t_1 )</th>
<th>( x_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40.0^\circ</td>
<td>1.780 s</td>
<td>16.36 m</td>
</tr>
<tr>
<td>42.5^\circ</td>
<td>1.853 s</td>
<td>16.39 m</td>
</tr>
<tr>
<td>45.0^\circ</td>
<td>1.923 s</td>
<td>16.31 m</td>
</tr>
<tr>
<td>47.5^\circ</td>
<td>1.990 s</td>
<td>16.13 m</td>
</tr>
</tbody>
</table>

Maximum distance is achieved at \( \theta = 42.5^\circ \).

Assess: The well-known “fact” that maximum distance is achieved at \( 45^\circ \) is true only when the projectile is launched and lands at the same height. That isn’t true here. The extra 0.03 m = 3 cm obtained by increasing the angle from 40.0^\circ to 42.5^\circ could easily mean the difference between first and second place in a world-class meet.
6.38. Model: Apply the particle model and the constant-acceleration equations of kinematics.

Visualize:

Solve: For the football:

\[ y_{IF} = y_{OF} + (v_{yF})t_{IF} + \frac{1}{2}(a_t)t_{IF}^2 \]

\[ \Rightarrow 0 = 0 + (v_{yF} \sin 40^\circ)t_{IF} - \frac{1}{2}(g)t_{IF}^2 \Rightarrow t_{IF} = 0 \text{ m/s and } t_{IF} = \frac{2v_{OF} \sin 40^\circ}{g} \]

Thus

\[ x_{IF} = x_{OF} + (v_{xF})t_{IF} + \frac{1}{2}(a_x)t_{IF}^2 = 0 + (v_{xF} \cos 40^\circ)t_{IF} + 0 \]

\[ = (v_{xF} \cos 40^\circ)\frac{2v_{OF} \sin 40^\circ}{g} = \frac{2v_{OF}^2}{g} \cos 40^\circ \]

For Doug:

\[ x_{ID} = x_{OD} + (v_{xD})t_{ID} + \frac{1}{2}(a_D)t_{ID}^2 = 20 \text{ m} + (6.0 \text{ m/s})t_{ID} + 0 \text{ m} = 20 \text{ m} + (6.0 \text{ m/s})\frac{2v_{OF}^2}{g} \sin 40^\circ \]

Since Doug and the football meet together, \( x_{IF} = x_{ID} \). Equating the expressions for Doug and the football yields:

\[ \frac{2v_{OF}^2}{g} \sin 40^\circ \cos 40^\circ = 20 \text{ m} + (6.0 \text{ m/s})\frac{2v_{OF}^2}{g} \sin 40^\circ \]

\[ \Rightarrow 0.1 v_{OF}^2 - (0.787 \text{ m/s}) v_{OF} - (20 \text{ m/s}^2) = 0 \Rightarrow v_{OF} = 18.6 \text{ m/s and } -10.7 \text{ m/s} \]

The negative solution is unphysical.

Assess: A speed of 18.6 m/s or 42 mph for the ball is reasonable. The mass of the ball, which was given in the problem, was not needed.
Homework 6. Problem 6:

A 1.30 kg toaster is not plugged in. The coefficient of static friction between the toaster and a horizontal countertop is 0.35. To make the toaster start moving, you carelessly pull on its electric cord. (a) For the cord tension to be as small as possible, you should pull at what angle above the horizontal? (b) With this angle, how large must the tension be?

With motion impending
\[ n + T \sin \theta = mg = 0 \]
\[ n = mg - T \sin \theta \]
\[ f = \mu_s n = \mu_s (mg - T \sin \theta) \]
and
\[ T \cos \theta - \mu_s mg + \mu_s T \sin \theta = 0 \]
\[ T \cos \theta + \mu_s T \sin \theta = \mu_s mg \]

\[ T = \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta} \]

To minimize \( T \), we maximize \( \cos \theta + \mu_s \sin \theta \)

\[ \frac{d}{d\theta} (\cos \theta + \mu_s \sin \theta) = 0 = -\sin \theta + \mu_s \cos \theta \]

\[ -\sin \theta = -\mu_s \cos \theta \]
\[ \mu_s = \frac{\sin \theta}{\cos \theta} = \tan \theta \]
\[ \theta = \tan^{-1} \mu_s \]

a) \( \theta = \tan^{-1} \mu_s = \tan^{-1} (0.350) = 19.3^\circ \)

b) \[ T = \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta} = \frac{(0.350)(1.30 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 19.3^\circ + 0.350 \sin 19.3^\circ} = 4.21 \text{ N} \]