8.28. Model: Blocks 1 and 2 are our systems of interest and will be treated as particles. Assume a frictionless rope and massless pulley.

Visualize:

Known:
- \( m_1 = 1 \) kg
- \( m_2 = 2 \) kg
- \( \mu_k = 0.30 \)
- \( T_{\text{pull}} = 20 \) N

Find:
- \( a_2 \)

Acceleration constraint:
- \( a_2 = a = -a_1 \)

Solve:
The blocks accelerate with the same magnitude but in opposite directions. Thus the acceleration constraint is \( a_2 = a = -a_1 \), where \( a \) will have a positive value. There are two real action/reaction pairs. The two tension forces will act as if they are action/reaction pairs because we are assuming a massless rope and a frictionless pulley. Make sure you understand why the friction forces point in the directions shown in the free-body diagrams, especially force \( f_1' \) exerted on block 2 by block 1. We have quite a few pieces of information to include. First, Newton’s second law for blocks 1 and 2:

\[
\begin{align*}
(F_{\text{net on 1}})_x &= f_1 - T_1 = \mu_k n_1 - T_1 = m_1 a_1 = -m_1 a \\
(F_{\text{net on 1}})_y &= n_1 - m_1 g = 0 \Rightarrow n_1 = m_1 g \\
(F_{\text{net on 2}})_x &= T_{\text{pull}} - f_2 - T_2 = T_{\text{pull}} - f_1' - \mu_k n_2 - T_2 = m_2 a_2 = m_2 a \\
(F_{\text{net on 2}})_y &= n_2 - n_1' - m_2 g = 0 \Rightarrow n_2 = n_1' + m_2 g
\end{align*}
\]

We’ve already used the kinetic friction model in both \( x \)-equations. Next, Newton’s third law:

\[
\begin{align*}
n_1' &= n_1 = m_1 g \\
f_1' &= f_1 = \mu_k n_1 = \mu_k m_1 g \\
T_1 &= T_2 = T
\end{align*}
\]

Knowing \( n_1' \), we can now use the \( y \)-equation of block 2 to find \( n_2 \). Substitute all these pieces into the two \( x \)-equations, and we end up with two equations in two unknowns:

\[
\begin{align*}
\mu_k m_1 g - T &= -m_1 a \\
T_{\text{pull}} - T - \mu_k m_1 g - \mu_k (m_1 + m_2) g &= m_1 a
\end{align*}
\]

Subtract the first equation from the second to get

\[
T_{\text{pull}} - \mu_k (3m_1 + m_2) g = (m_1 + m_2) a \Rightarrow a = \frac{T_{\text{pull}} - \mu_k (3m_1 + m_2) g}{m_1 + m_2} = 1.77 \text{ m/s}^2
\]
8.34. **Model:** Use the particle model for the block of mass $M$ and the two massless pulleys. Additionally, the rope is massless and the pulleys are frictionless. The block is kept in place by an applied force $F$.

**Visualize:** Pictorial representation

**Solve:** Since there is no friction on the pulleys, $T_1 = T_3$ and $T_2 = T_5$. Newton’s second law for mass $M$ is

$$T_1 - w = 0 \text{ N} \Rightarrow T_1 = Mg = (10.2 \text{ kg})(9.8 \text{ m/s}^2) = 100 \text{ N}$$

Newton’s second law for the small pulley is

$$T_2 + T_1 - T_3 = 0 \text{ N} \Rightarrow T_2 = T_3 = \frac{T_1}{2} = 50 \text{ N} = T_5 = F$$

Newton’s second law for the large pulley is

$$T_4 - T_2 - T_3 - T_5 = 0 \text{ N} \Rightarrow T_4 = T_2 + T_3 + T_5 = 150 \text{ N}$$
8.38. **Model:** Assume the particle model for the book (B) and the coffee cup (C), the models of kinetic and static friction, and the constant-acceleration kinematic equations.

**Visualize:** Pictorial representation

![Diagram of book and coffee cup with forces labeled](image)

**Physical representation**

![Diagram showing forces on book](image)

**Coffee cup**

**Stationary book**

**Solve:** (a) Using \( v_{ix}^2 = v_{ix}^2 + 2a(x_i - x_u) \),

\[
0 \text{ m}^2/\text{s}^2 = (3.0 \text{ m/s})^2 + 2a(x_i) \Rightarrow ax_i = -4.5 \text{ m}^2/\text{s}^2
\]

To find \( x_i \), we must first find \( a \). Newton’s second law for the book and the coffee cup is

\[
\sum F_m = n_b - w_b \cos 20^\circ = 0 \text{ N} \Rightarrow n_b = (1.0 \text{ kg}) (9.8 \text{ m/s}^2) \cos 20^\circ = 9.21 \text{ N}
\]

\[
\sum F_m = -T - f_k - w_b \sin 20^\circ = m_b a_b \quad \sum F_m = -w_C = m_c a_c
\]

The last two equations can be rewritten, using \( a_c = a_b = a \), as

\[
-T - \mu_k n_b - m_b g \sin 20^\circ = m_b a_b \quad T - m_c g = m_c a_c
\]

Adding the two equations,

\[
a(m_c + m_b) = -g(m_c + m_b \sin 20^\circ) - \mu_k (9.21 \text{ N})
\]

\[
\Rightarrow (1.5 \text{ kg}) a = -((9.8 \text{ m/s}^2)(0.5 \text{ kg} + (1.0 \text{ kg}) \sin 20^\circ) - (0.20)(9.21 \text{ N}) \Rightarrow a = -6.73 \text{ m/s}^2
\]

Using this value for \( a \), we can now find \( x_i \) as follows:

\[
x_i = \frac{-4.5 \text{ m}^2/\text{s}^2}{a} = \frac{-4.5 \text{ m}^2/\text{s}^2}{-6.73 \text{ m/s}^2} = 0.669 \text{ m}
\]
(b) The maximum static friction force is $f_{s,\text{max}} = \mu B m_b = (0.50)(9.21 \text{ N}) = 4.60 \text{ N}$. We’ll see if the force $f_s$ needed to keep the book in place is larger or smaller than $f_{s,\text{max}}$. When the cup is at rest, the string tension is $T = m_c g$. Newton’s first law for the book is

$$\sum (F_{\text{on b}})_s = f_s - T - w_b \sin 20^\circ = f_s - m_c g - m_b g \sin 20^\circ = 0$$

$$\Rightarrow f_s = (M_c + M_b \sin 20^\circ) g = 8.25 \text{ N}$$

Because $f_s > f_{s,\text{max}}$, the book slides back down.
8.40. **Model:** Use the particle model for the two blocks. Assume a massless rope, and massless, frictionless pulleys.

**Visualize:**

Note that for every meter block 1 moves forward, one meter is provided to block 2. So each rope on $m_2$ has to be lengthened by one-half meter. Thus the acceleration constraint is $a_2 = -\frac{1}{2}a_1$.

**Solve:** Newton’s second law for block 1 is $T = m_1a_1$. Newton’s second law for block 2 is $2T - w_2 = m_2a_2$. Combining these two equations gives

$$2(m_1a_1) - m_2g = m_1\left(-\frac{1}{2}a_1\right) \Rightarrow a_1(4m_1 + m_2) = 2m_2g \Rightarrow a_1 = \frac{2m_2g}{4m_1 + m_2}$$

where we have used $a_2 = -\frac{1}{2}a_1$.

**Assess:** If $m_1 = 0$ kg, then $a_2 = -g$. This is what is expected for a freely falling object.
9.8. Model: Use the particle model for the sled, the model of kinetic friction, and the impulse-momentum theorem.

Visualize:

Note that the force of kinetic friction \( f_k \) imparts a negative impulse to the sled.

Solve: Using \( \Delta \mathbf{p} = J_x \), we have

\[
p_{0x} - p_{fx} = \int F_x(t) dt = -f_k \int_{t_i}^{t_f} dt = -f_k \Delta t \Rightarrow mv_{ix} - mv_{fx} = -\mu_k n \Delta t = -\mu_k mg \Delta t
\]

We have used the model of kinetic friction \( f_k = \mu_k n \), where \( \mu_k \) is the coefficient of kinetic friction and \( n \) is the normal (contact) force by the surface. The force of kinetic friction is independent of time and was therefore taken out of the impulse integral. Thus,

\[
\Delta t = \frac{1}{\mu_k g} (v_{fx} - v_{ix}) = \frac{1}{(0.25)(9.8 \text{ m/s}^2)} (8.0 \text{ m/s} - 5.0 \text{ m/s}) = 1.22 \text{ s}
\]
9.14. **Model:** Choose car + rainwater to be the system.

**Visualize:** There are no external horizontal forces on the car + water system, so the horizontal momentum is conserved.

**Solve:** Conservation of momentum is \( p_i = p_f \). Hence,

\[
(\text{car} + \text{water})(20 \text{ m/s}) = (\text{car})(22 \text{ m/s}) + (\text{water})(0 \text{ m/s})
\]

\[
\Rightarrow (5000 \text{ kg} + \text{water})(20 \text{ m/s}) = (5000 \text{ kg})(22 \text{ m/s}) \Rightarrow \text{water} = 500 \text{ kg}
\]
9.20. **Model:** Because of external friction and drag forces, the car and the blob of sticky clay are not exactly an isolated system. But during the collision, friction and drag are not going to be significant. The momentum of the system will be conserved in the collision, within the impulse approximation.

**Visualize:**

**Pictorial representation**

Before

\[ m_B = 10 \text{ kg} \]

\[ (v_{iB})_x = \] (not shown)

\[ (v_{iC})_x = -2.0 \text{ m/s} \]

\[ m_C = 1500 \text{ kg} \]

\[ m_B \] (not shown)

\[ v_{iC} = 0 \]

\[ v_{fC} = \] (not shown)

After

\[ x \]

\[ v_{fB} = \] (not shown)

\[ x \]

**Solve:** The conservation of momentum equation \( p_i = p_f \) is

\[
(m_C + m_B)(v_x)_s = m_B(v_{iB})_x + m_C(v_{iC})_x
\]

\[
\Rightarrow 0 \text{ kg m/s} = (10 \text{ kg})(v_{iB})_x + (1500 \text{ kg})(-2.0 \text{ m/s}) \Rightarrow (v_{iB})_x = 300 \text{ m/s}
\]

**Assess:** This speed of the blob is around 600 mph which is very large. However, we must point out that a very large speed is *expected* in order to stop a car with only 10 kg of clay.
### Model

We will model the two fragments of the rocket after the explosion as particles. We assume the explosion separates the two parts in a vertical manner. This is a three-part problem. In the first part, we will use kinematic equations to find the vertical position where the rocket breaks into two pieces. In the second part, we will apply conservation of momentum to the system (that is, the two fragments) in the explosion. In the third part, we will again use kinematic equations to find the velocity of the heavier fragment just after the explosion.

#### Visualize

![Diagram of rocket fragments](image)

#### Solve

The rocket accelerates for 2.0 s from rest, so

\[
v_{x} = v_{0y} + a_{y}(t_{1} - t_{0}) = 0 \text{ m/s} + (10 \text{ m/s}^{2})(2 \text{ s} - 0 \text{ s}) = 20 \text{ m/s}
\]

\[
y_{1} = y_{0} + v_{0y}(t_{1} - t_{0}) + \frac{1}{2}a_{y}(t_{1} - t_{0})^{2} = 0 + 0 + \frac{1}{2}(10 \text{ m/s}^{2})(2 \text{ s})^{2} = 20 \text{ m}
\]

At the explosion the equation \(p_{f} = p_{i}\) is

\[
m_{L}v_{2yL} + m_{H}v_{2yH} = (m_{L} + m_{H})v_{1y} \Rightarrow (500 \text{ kg})(v_{2yL})_{L} + (1000 \text{ kg})(v_{2yH})_{H} = (1500 \text{ kg})(20 \text{ m/s})
\]

To find \((v_{2yH})_{H}\) we must first find \((v_{2yL})_{L}\), the velocity after the explosion of the upper section. Using kinematics,

\[
(v_{y})_{L}^{2} = (v_{y})_{L}^{2} + 2(-9.8 \text{ m/s}^{2})(y_{ML} - y_{2L}) \Rightarrow (v_{y})_{L} = \sqrt{2(9.8 \text{ m/s}^{2})(530 \text{ m} - 20 \text{ m})} = 99.98 \text{ m/s}
\]

Now, going back to the momentum conservation equation we get

\[
(500 \text{ kg})(99.98 \text{ m/s}) + (1000 \text{ kg})(v_{2yH})_{H} = (1500 \text{ kg})(20 \text{ m/s}) \Rightarrow (v_{2yH})_{H} = -20.0 \text{ m/s}
\]

The negative sign indicates downward motion.