CIRCULAR ORBITS

UNDER FLAT EARTH APPROXIMATION
MAXIMUM RANGE OF A PROJECTILE DEPENDS ON THE MAGNITUDE OF \( R \)

\[ \theta \]

IF EARTH HAS CURVED SURFACE AND \( \theta \) IS SUFFICIENTLY LARGE AT SUFFICIENTLY HIGH \( h \), THEN THE TRAJECTORY OF THE PROJECTILE WOULD BE AN ORBIT.

FOR PROJECTILES THE ONLY FORCE ACTING WOULD BE GRAVITATIONAL \( (\vec{N}) \)

\[ \sum F_x = m a_x = |\vec{N}| \]

\[ \frac{m v^2}{R} = mg \Rightarrow v_{orbit} = \sqrt{Rg} \]

*IF PROJECTILE IS LAUNCHED WITH \( v_{orbit} \)
THE ONLY FORCE WOULD BE \( \vec{N} \)
**Period (T) of the Orbit**

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\frac{2\pi r}{T}} = \frac{2\pi R}{2\pi} = \frac{2\pi R}{2\pi} = \frac{R}{r} = \frac{2\pi \sqrt{\frac{R}{g}}}{2\pi} = \sqrt{\frac{R}{g}}$$

**What is the Period of Moon's Orbit?**

Radius of Moon's Orbit = $3.84 \times 10^8$ m

$$T = 2\pi \sqrt{\frac{3.84 \times 10^8}{9.80}} \approx 31.25 \text{ hrs}$$

*But we know from astronomical data that $T$ is about 1 day.*

**Contradiction**

$$g_{\text{moon}} = \frac{f^2}{T^2} \frac{4\pi^2 R^2}{2 \pi \text{ days}^2} = \frac{4\pi (3.84 \times 10^8)^2}{2 \pi \text{ days}^2} \approx 0.002 \text{ m/s}^2$$

$T_{\text{moon}} \approx 27 \text{ days}$
CENTRIFUGAL FORCE

CENTRIFUGAL FORCE IS A FICTITIOUS* FORCE THAT ACTS IN OPPOSITE DIRECTION TO THE CENTER SEEKING OR CENTRIPETAL FORCE.

* FICTITIOUS BECAUSE IT IS NOT INCLUDED IN THE FREE BODY DIAGRAM

* ITS EXISTENCE RELIES ON THE LAW OF INERTIA (NEWTON'S FIRST LAW)

* THERE IS NO AGENT EXERTING THE CENTRIFUGAL FORCE — HENCE FICTITIOUS

* (PREP) NEWTON'S FIRST LAW, WHEN THERE IS NO EXTERNAL FORCE .... SO WE WILL BE WORKING WITH NON-INERTIAL REFERENCE FRAMES

\[
P_{\text{inertial (from ground)}} \quad \text{NON-INERTIAL (FROM PASSENGER)}
\]

\[
P_{\text{accelerating}} \quad P_{\text{decelerating}}
\]
CAR TURNING A CURVE

**CENTRIPETAL:**

**THE NORMAL FORCE COMPONENT IN RADIAL DIRECTION**

**CENTRIFUGAL**

*Obeying Newton's First Law* 
*Car* (when no force is acting) *tends to move along straight line*

The car door pushes in the normal direction for keeping the turn

**Example:** Roller Coaster

**Note:** When a car takes a sharp turn you feel as if you are "thrown" away from the center of the curve. There is no such real force 😊
USEFULNESS OF CENTRIFUGAL FORCE

EX: ROLLER COASTER

AT THE BOTTOM:

\[ \text{n} - \text{w} = \text{F}_\text{net} = m \text{a}_\text{x} \]

\[ \therefore \text{n} = m \frac{v^2}{\text{r}_\text{bottom}} + \text{w} \text{time} = \text{w}_{\text{app}} \]

AT THE TOP

\[ \text{n} + \text{w} = m \text{a}_\text{x} = m \frac{v^2}{\text{r}_\text{top}} \]

\[ \text{n} = m \frac{v^2}{\text{r}_\text{top}} - \text{w} \text{time} = \text{w}_{\text{app}} \]

OBSERVE THAT WEIGHT AT THE TOP IS SMALLER THAN THE WEIGHT AT THE BOTTOM
AT THE TOP:

\[ n + w_{\text{true}} = m a_n = m \frac{v^2}{\rho} \]

\[ n = \frac{m v^2}{\rho} - w_{\text{true}} \]

IS THE NORMAL FORCE TRACK EXERTS ON THE CAR.

If \( \rho_{\text{top}} \) is sufficiently large,

The apparent weight will be large enough such that \( n \) app \( \approx \) \( n \) true.

But, if \( \rho_{\text{top}} \) decreases \( \frac{m v^2}{\rho_{\text{top}}} \rightarrow 0 \)

\( \therefore \) no centripetal force due to the track but,

the weight of the car would be enough to bring the car down.

Critical Speed \( v_c \) (at which \( \rho = \infty \))

\[ \frac{m v^2}{\rho} = mg \Rightarrow v_c = \sqrt{\frac{mg}{m}} = \sqrt{rg} \]
Critical Angular Velocity \( (\omega_c) \)

Critical angular velocity is the angular velocity at which weight alone provides the necessary centripetal acceleration.

\[
\begin{align*}
\omega < \omega_c & \quad \Rightarrow \quad \text{Loss of orbit} \\
\omega = \omega_c & \quad \Rightarrow \quad \text{Weight provides centripetal acceleration} \\
\omega > \omega_c & \quad \Rightarrow \quad \text{Object is in orbit}
\end{align*}
\]

\[
\omega_c = \sqrt{\frac{g}{r}}
\]

\[
\omega_e = \omega_c \cdot r = \frac{2\pi r}{T} = \frac{2\pi r}{2\pi \sqrt{\frac{g}{r}}} = \sqrt{\frac{g}{r}}
\]
NON-UNIFORM CIRCULAR MOTION

\[ \Sigma F_r = m a_r = \frac{m v_r^2}{r} \]

\[ \Sigma F_T = ma_T \]

\[ \Sigma F = 0 \]

\[ a = a_r + a_T \]

\[ \phi = \tan^{-1} \left( \frac{a_T}{a_r} \right) \]

\[ a = \sqrt{a_r^2 + a_T^2} \]

Eqs. of Motion:

1. \[ s_f = s_i + v_i t + \frac{1}{2} a_i (at)^2 \]

2. \[ s_f = s_i + v_i t + \frac{1}{2} a_T (at)^2 \]

3. \[ \theta_f = \theta_i + \omega_i t + \frac{1}{2} \omega (at)^2 \]

4. \[ \omega_f = \omega_i + \omega_T t \]

5. \[ \omega_f = \omega_i + \omega_T (at) \]
READ

SECTION 7.5

CHAPTER #8

SECTIONS 8.1, 8.2