If height of the rail is known what would be the velocity at the bottom?

Does velocity of the car depend on the shape of the track?

Ballistic Pendulum

Are there any situations where we need more info to solve?
FREE FALL REVISITED

UNDER THE FORCE OF GRAVITY

\[ \frac{v_f^2}{2} = v_i^2 + 2g y \Delta y \]

\[ = v_i^2 - 2g (y_f - y_i) \]

\[ v_f^2 + 2gy_f = v_i^2 + 2gy_i \]

IN GENERAL

\[ v^2 + 2gy = \text{CONSTANT} \]

\[ v^2 + 2gy \text{ IS CONSERVED} \]
MORE FORMAL APPROACH

FROM NEWTON'S SECOND LAW (ALONG VERTICAL DIRECTION)

\[ (F_{\text{net}})_y = m \frac{dy}{dt} = m \frac{dv_y}{dt} \]

CHAIN RULE:

\[ \frac{dv_y}{dt} = \frac{dv_y}{dy} \cdot \frac{dy}{dt} \]

\[ (F_{\text{net}})_y = -mg \]

\[ m \frac{dy}{dt} = -mg \]

\[ \int m \frac{dy}{dy} = -\int mg \, dy \]

\[ \frac{1}{2} m [v_f^2 - v_i^2] = -mg [y_f - y_i] \]
\[ \frac{1}{2} mv_f^2 + mg y_f = \frac{1}{2} mu_i^2 + mg y_i \]

**In General**

\[ \frac{1}{2} mu^2 + mg y = \text{Constant} \]

\[ K + U = \text{Constant} \]

**Where**

\[ K = \frac{1}{2} mu^2, \text{Kinetic Energy} \]

[ Energy of Motion ]

\[ U = mg y, \text{Potential Energy} \]

[ Energy of Position ]

**Total Energy = Potential Energy + Kinetic Energy**
(a) \[ v = 0 \]

\[ \begin{align*}
2 & \quad 0 \\
1 & \quad O \\
& \quad \text{GND}
\end{align*} \]

**RANK THEM IN THE ORDER OF THEIR POTENTIAL ENERGY**

(b) \[ k_\text{e}^{-} \quad v_0 \quad \text{v} \]

\[ h \]

**FIND** \( v \), **if** \( h = 5.0 \text{ m} \), \( v_0 = 2.0 \text{ m/s} \)
MOTION ALONG UN Even SURFACES

CAN YOU EMPLOY LINEAR CIRCULAR EQUATIONS OF MOTION?

IS THERE A SIMPLER WAY?

NOTICE THAT X-AXIS IS REPRESENTED BY S (TO BE COMPAREABLE WITH TEXT BOOK)
\[(F_{\text{net}})_s : -mg \sin \theta = ma_s\]
\[(F_{\text{net}})_y : n - mg \cos \theta = 0\]

\[(F_{\text{net}})_s = ma_s = m \frac{d^2 s}{dt^2}\]

\[m \frac{dv_s}{ds} \frac{ds}{dt} = -mg \sin \theta\]

\[m v_s \frac{dv_s}{ds} = -mg \sin \theta ds\]

\[\therefore m v_s \frac{dv_s}{ds} = -mg \, dy\]

Integrating

\[\int_{v_i}^{v_f} m v_s \, dv_s = -\int_{y_i}^{y_f} mg \, dy\]

\[\sin \theta = \frac{y}{ds}\]

\[\frac{1}{2} m (v_f^2 - v_i^2) = -mg (y_f - y_i) ds \sin \theta \, dy\]

\[\frac{1}{2} m v_f^2 + mg y_f = \frac{1}{2} m v_i^2 + mg y_i\]
PRACTICAL APPLICATION OF CONSERVATION OF ENERGY

(i) Ballistic Pendulum

A 10 g bullet is fired with speed $v_b$ into a wooden block of mass 1200 g that hangs from a 150 cm long string. If bullet embeds into block and block swings out an angle $\theta = 40^\circ$, what is the speed of the bullet? $v_b =$ ?

$m = 0.01 \text{ kg}$
$m = 1.20 \text{ kg}$

$v_w = 0$
PART (1)

APPLY CONSERVATION OF MOMENTUM TO FIND $v_b$

$$m v_b + M v_n^0 = (m + M) v$$

$$m v_b = (m + M) v$$

$$v_b = \frac{(m + M) v}{m}$$

PART (2) APPLY CONSERVATION OF ENERGY

$$\frac{1}{2} (m + M) v^2 + 0 = \frac{1}{2} (m + M) v_f^2 + (m + M) h' g$$

$$v^2 = 2 h' g$$

$$v^2 = 2 h' g$$

$$k =$$

$$h = h_0 + h'$$

$$h' = h - h_0 \cos \theta$$

$$v_b = \frac{m + M}{m} \left(2 h' g\right)^{1/2}$$