MECHANICAL ENERGY

\[ E = K + U \]

CONSERVATION OF \( E_{\text{mech}} \):

\[ K + U = \text{constant} \]

\[ \Rightarrow \Delta E = \Delta K + \Delta U = 0 \]

EX: SPRING, RUBBER ETC.

WHEN IS \( E_{\text{mech}} \) CONSERVED?
WHAT HAPPENS IF \( E_{\text{mech}} \) IS NOT CONSERVED?

\( E_{\text{mech}} \) IS CONSERVED IN ELASTIC SYSTEMS
EX: SPRING

\[ F_{\text{sp}} \propto AS \]

\[ F_{\text{sp}} = kAS \]

\( k \): IS SPRING CONSTANT

\[ k = \frac{F_{\text{sp}}}{AS} : N/m \]
$K$ is the slope
of $F_{sp}$ vs. $\Delta s$ graph.

$\Delta s$, and $F_{sp}$ are vectors. So more mathematically:

$$F_{sp} = -K \Delta s$$  - Hook's Law

$F_{sp}$ is always opposite to displacement.

Hook's Law holds good in the limit of elasticity.

That is,

If the spring is stretched too much / compressed too much, Hook's Law may not be valid.
A block of 2 kg mass is pulled by a toy car through a spring of spring constant $k = 50 \, N/m$. If the toy car moves at 5.0 cm/s, when does the block slip? $\mu_s = 0.6$ between block and surface.

Diagram:

- $2.0 \, kg$ block
- Spring of constant $k$
- Forces $F_{SP}$, $F_s$, and $mg$

Equations:

1. $(F_{net})_x : F_{SP} = F_s$
2. $(F_{net})_y : n = mg$
3. $M_s \, mg = k \, \Delta x$

Thus, $\Delta x = \frac{M_s \, mg}{k} = \frac{0.6 \times 2 \, kg \times 9.8}{50}$

Time taken for the block to slip in:

$$\frac{\Delta x}{v_i} = \frac{23.5}{5} = 4.75 \, \text{s}$$
Elastic Potential Energy

The restoring force in a spring depends on the displacement.

When the spring is compressed, the P.E is "stored" in the spring.
When the spring is expanded, the stored P.E is converted into K.E.

\[ U_s = \frac{1}{2} k (\Delta s)^2 \]

\[ U_{si} = \frac{1}{2} k (\Delta s_i)^2 = \frac{1}{2} k (s_i - s_0)^2 \]

\[ U_{sf} = \frac{1}{2} k (\Delta s_f)^2 = \frac{1}{2} k (s_f - s_0)^2 \]

Note: Elastic P.E is independent of the direction of displacement.
ELASTIC COLLISIONS

A COLLISION IS SAID TO BE PERFECTLY ELASTIC IF $E_{\text{mech}}$ IS CONSERVED

COLLISION MUST OBEY

(i) CONSERVATION OF MOMENTUM

(ii) CONSERVATION OF ENERGY

BEFORE

\[
\begin{align*}
\text{(1)} & \quad (v_{ix})_1 \quad (v_{ix})_2 = 0 \\
\text{(2)} & \quad m_1 \quad m_2
\end{align*}
\]

DURING

\[
\begin{align*}
\text{(1)} & \quad (v_{ix})_1' + 0 = m_1 (v_{fx})_1 + m_2 (v_{fx})_2 \\
\text{(2)} & \quad \text{Figures}
\end{align*}
\]

AFTER

\[
\begin{align*}
\text{(1)} & \quad (v_{fx})_1' = \frac{m_1 (v_{ix})_1 - m_2 (v_{fx})_2}{m_1} \\
\text{(2)} & \quad \frac{1}{2} m_1 (v_{ix})^2 + 0 = \frac{1}{2} m_1 (v_{fx})^2 + \frac{1}{2} m_2 (v_{fx})^2 \\
\text{Substitute Eq. 1 into (ii) and reorganize}
\end{align*}
\]

\[
(\frac{m_2}{m_1} (v_{fx})_2 - 2 (v_{ix})_1) = 0
\]
\[(v_{fx})_2 = 0 \quad \text{or} \quad (1 + \frac{m_2}{m_1})(v_{fx})_2 = 2(v_{ix})_1\]

\[\therefore (v_{fx})_2 = \frac{2m_1}{m_1+m_2} (v_{ix})_1\]

Plug in this result in Eqn. 1 to get

\[(v_{fx})_1 = \frac{m_1-m_2}{m_1+m_2} (v_{ix})_1\]

\[(v_{fx})_1, (v_{fx})_2 \quad \text{are velocities after collision.}\]
REBOUNDING PENDULUM (P. 289)

A 200 g steel ball hangs on a 1.0 m long string. If the ball is suspended by 45° from vertical position and released and if it hits a block at bottom most point what is the angle of rebound. The mass of the block is 500 g.

This problem has 3 parts:
Part 1: Conservation of energy
Part 2: " of momentum
Part 3: " of energy

\[ v' = \sqrt{\frac{1}{2} m v_i^2 + mg \Delta x} = \sqrt{\frac{1}{2} mv_f^2} + 0 \]

Find \( v_f \) of the ball before collision

\[ mg \Delta x (1 - \cos 45°) = \frac{1}{2} m v_f^2 \]
Part 2:

Law of Conservation of Momentum

\[ m_1(v_{ix})_1 + 0 = m_1(v_{fx})_1 + m_2(v_{fx})_2 \]

Using result of elastic collision

\[ (v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{ix})_1 \]

\[ = \frac{0.2 - 0.5}{0.7} (2.40) \equiv 0.90 \]

\[ = -1.03 \, \text{m/s} \]

\[ (v_{fx})_2 = \frac{2m_1}{m_1 + m_2} (v_{ix})_1 \]

\[ = \frac{0.4}{0.7} (2.40) \]

\[ = 1.37 \, \text{m/s} \]
PART 3: CONSERVATION OF ENERGY FOR CALCULATING REBOUND

\[(v_{ix})_1 = (v_{fx})_2\]

\[\frac{1}{2} m (v_{ix})^2 + mgh = \frac{1}{2} m (v_{fx})^2 + mgh\]

\[\frac{1}{2} \times m (-1.03)^2 = (0, g x (1 - \cos \theta))\]

\[1 - \cos \theta_1 = \frac{(-1.03)^2}{2g x}\]

\[\cos \theta_1 = 1 - \frac{(-1.03)^2}{2g x}\]

\[\theta_1 = \cos^{-1}(0.94587) = 18.9^\circ\]

STEEL BALL REBOUNDS BY 18.9^\circ AFTER COLLISION