Example (P.57)

A rocket sled accelerates at 50 m/s² for 5.0 s, coasts for 3.0 s, decelerates at 3.0 m/s² until completely stops.

(i) Find maximum vel.
(ii) Total distance traveled?

Kinematic Equations of Motion are:

\[ v_f = v_i + a \Delta t \]
\[ s_f = s_i + \frac{1}{2} a (\Delta t)^2 + v_i \Delta t \]
\[ v_f^2 = v_i^2 + 2a s_f \]
MAX. VELOCITY CORRESPONDS TO MAX. ACCELERATION

START: \( v_i = 0 \), \( s_i = 0 \), \( t = 0 \)

\( a \) = \( 50.0 \text{ m/s}^2 \)

\( t = 5 \text{ s} \) \( v_f = ? \), \( s_f = ? \), \( t = 5 \)

\( \Delta t = 5 \)

\[ v_f = v_i + a \Delta t \]

\[ = 0 + (50)(5) \]

Max. vel = 125 m/s

---

250
TOTAL DISTANCE COVERED

= DISTANCE BETWEEN (START, COAST)

+ DISTANCE DURING COAST

+ DISTANCE UNTIL STOPS

Part 1: \( v_i = 0 \), \( t_i = 0 \), \( S_i = 0 \), \( S_f = ? \)

\( a = 50 \text{ m/s}^2 \), \( t_f = 5 \text{ s} \)

\[
(S_f)_1 = S_i + \frac{1}{2} a (t_f)^2 + v_i \Delta t
\]

\[
= \frac{1}{2} (50)(5)^2
\]

\[
= 625 \text{ m}
\]

Part 2: \( a = 0 \), \( \Delta t = 3 \text{ s} \), \( S_i = 0 \)

\( v_i = v_f \) of Part 1 = 250 m/s

\[
(S_f)_2 = S_i + v_i \Delta t
\]

\[
= 0 + 250 \times 3 = 750 \text{ m}
\]

Part 3: \( a = -3.0 \text{ m/s}^2 \), \( v_f = 0 \), \( S_i = 250 \text{ m/s} \)

\( v_i = 250 \text{ m/s} \)

\[
(S_f)_3 = S_i + \frac{v_f^2 - v_i^2}{2a}
\]

\[
= 0 + \frac{(250)^2}{2(-6)}
\]
FREE FALL

Motion of an object under the influence of gravity only, and no other forces

When air resistance is zero all objects irrespective of their masses hit the ground at the same time, when dropped from same height

In free fall, objects will experience the same acceleration due to gravity of

\[ g = 9.8 \text{ m/s}^2 \], vertically downward

\[ g \] is the magnitude of \( \vec{a} \) for free fall
Example:

A cannonball is fired up with an initial speed of 100 m/s. How high does it go?

\[ v_i = 100 \text{ m/s} \]
\[ v_f = 0 \]
\[ a = -g \]
\[ \Delta s = ? \]

\[ v_f^2 = v_i^2 + 2a\Delta s \]
\[ 0 = (100)^2 - 2g\Delta s \]
\[ \Delta s = \frac{(100)^2}{2g} = \frac{10^4}{19.6} \]
\[ \approx 510 \text{ m} \]
**Motion in an Inclined Plane**

\[ \vec{a}_{ff} = \vec{a}_{\text{free fall}} \]

- **Surface is frictionless**
- **Object is constrained to move on the incline**

\[ \vec{a}_{ff} = \vec{a}_f + \vec{a}_{\parallel} \]

\[ a_{\parallel} = \vec{a}_{ff} \sin \theta = \pm g \sin \theta \]

- **Case (i)** \( \theta = 0 \) \( a_{\parallel} = 0 \) **No sliding**
- **Case (ii)** \( \theta = 90^\circ \) \( a_{\parallel} = -g \) **Vertical fall**
- **Case (iii)** \( 0 < \theta < 90^\circ \) \( a_{\parallel} = g \sin \theta \)
Example:

A skier’s speed at the bottom of a 100 m long, frictionless snow-covered slope is 20 m/s. What is the angle of the slope?

\[ \Delta s = 100 \text{ m} \]
\[ v_i = 0 \]
\[ v_f = 20 \text{ m/s} \]
\[ a_s = g \sin \theta \]

\[ v_f^2 = v_i^2 + 2a \Delta s \]
\[ (20)^2 = 0 + 2 \times 9.8 \times 100 \sin \theta \]
\[ \sin \theta = \frac{400}{2 \times 9.8 \times 100} \approx 0.2 \]
\[ \theta = \sin^{-1}(0.2) \approx 11.8^\circ \]
INSTANTANEOUS ACCELERATION

The instantaneous acceleration \( \vec{a} \) at a specific instant of time \( t \) is the slope of the line that is tangent to the velocity-time graph.

\[
\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d \vec{v}}{dt}
\]

Instantaneous acceleration is the derivative of the velocity.
Extracting velocity from acceleration

1. Assume instantaneous acceleration
2. Divide $a_s = \frac{\Delta v}{\Delta t}$ graph into $N$ sub-intervals between $[t_i, t_f]$

We know, $\ddot{v}_f = \ddot{v}_i + \dot{a}_s \Delta t$

($\Delta v = v_f - v_i$)

$$\Delta v = \Delta v_1 + \Delta v_2 + \ldots + \Delta v_k + \ldots + \Delta v_N$$

$$= (a_s)_1 \Delta t + \ldots + (a_s)_N \Delta t$$

$$v_f - v_i = \sum_{k=1}^{N} (a_s)_k \Delta t$$

$$\therefore v_f = v_i + \int_{t_i}^{t_f} a_s \, dt$$
Example:

What would be the particle's velocity at $t = 8$ sec? If its initial velocity is $10 \text{ m/s}$, use the graph below.

\[ \Delta t = 4 \text{ s} \]
\[ a_s = 4 \text{ m/s}^2 \]
\[ v_i = 10 \text{ m/s} \]

\[ v_f = v_i + \text{area under } a_s - t \text{ graph} \]
\[ = v_i + \text{area of } 0 - \text{area of } 2 \]
\[ = v_i + a_s \Delta t + \frac{1}{2} a_s \Delta t \]
\[ = 10 + 4 \times 4 + \frac{1}{2} \times 4 \times 4 \]
\[ = 10 + 16 + 8 \]
\[ = 34 \text{ m/s} \]
TURNING POINTS

Let a particle's velocity be
\[ v_t = 10 - (t-5)^2 \text{ m/s} \]

(i) Find particle's acceleration.
(ii) Determine its turning points.

\[ a_t = \frac{dv_t}{dt} = \frac{d}{dt} (10 - (t-5)^2) \]
\[ = -2(t-5) \]
\[ = -2t + 10 \text{ m/s}^2 \]

At turning points, \( v_t = 0 \)

At turning point, velocity reverses the direction.

\[ 10 - (t-5)^2 = 0 \]
\[ t-5 = \pm \sqrt{10} = \pm 3.16 \]
\[ t = 5 \pm 3.16 \]
\[ = 8.33, 1.67 \]

\( t = 1.67, 8.33 \) are turning points for the velocity-time graph.

Velocity is zero at turning points.