GENERAL PRINCIPLES

Newton's Second Law
Expressed in $rtz$-component form:

$$ (F_{\text{net}})_r = \sum F_r = ma_r = \frac{mv^2}{r} = mw^2r $$

$$ (F_{\text{net}})_t = \sum F_t = \begin{cases} 0 & \text{uniform motion} \\ ma_t & \text{nonuniform motion} \end{cases} $$

$$ (F_{\text{net}})_z = \sum F_z = 0 $$

Uniform Circular Motion
- $v$ is constant.
- $\vec{F}_{\text{net}}$ points toward the center of the circle.
- The centripetal acceleration $\vec{a}$ points toward the center of the circle. It changes the particle's direction but not its speed.

Nonuniform Circular Motion
- $v$ changes.
- $\vec{a}$ is parallel to $\vec{F}_{\text{net}}$.
- The radial component $a_r$ changes the particle's direction.
- The tangential component $a_t$ changes the particle's speed.

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**IMPORTANT CONCEPTS**

**rtz-coordinates**

**Angular position**
\[ \theta = s/r \]

**Angular velocity**
\[ \omega = \frac{d\theta}{dt} \]
\[ v_t = \omega r \]
APPLICATIONS

Circular motion kinematics

Period \( T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \)

Uniform circular motion

\( v_i = \text{constant} \quad \omega = \text{constant} \)

\( \theta_i = \theta_i + \omega \Delta t \)

Nonuniform circular motion

\( \theta_i = \theta_i + \omega_i \Delta t + \frac{\alpha_i}{2} (\Delta t)^2 \)

\( \omega_e = \omega_i + \frac{\alpha_i}{r} \Delta t \)

Orbits

A circular orbit has radius \( r \) if

\( v = \sqrt{rg} \)

Apparent weight

Circular motion requires a net force pointing to the center. The apparent weight \( w_{app} = n \) is usually not the same as the true weight \( w \). \( n \) must be \( > 0 \) for the object to be in contact with a surface.
A 500 g ball swings in a vertical circle at the end of a 1.5 m long string. When the ball is at the bottom of the circle, the tension in the string is 15.0 N. What is the speed of the ball at that point?
\[ \Sigma F_r = T - mg = ma_r \]

\[ T - mg = \frac{mv_t^2}{r} \]

\[ 15 - (0.5)(9.80) = \frac{0.5v_t^2}{1.5} \]

\[ v_t = 5.50 \text{ m/s} \]
7.35  A concrete highway curve of radius 70 m is banked at 15° angle. What is the maximum speed limit with which a 1500 kg rubber-tired car can take this curve without sliding?

$$\mu_s (\text{rubber on concrete}) = 1.0$$
\[ \Sigma F_x = n \cos \theta - w - f_s \sin \theta = 0 \quad (1) \]
\[ \Sigma F_y = n \sin \theta + f_s \cos \theta = \frac{mv^2}{r} \quad (2) \]
\[ f_{s,\text{max}} = \mu_s n \]
\[ (2) \Rightarrow n \left( \sin \theta + \mu_s \cos \theta \right) = \frac{mv_{\text{max}}^2}{r} \]
\[ (1) \Rightarrow n \left( \cos \theta - \mu_s \sin \theta \right) = mg \]
\[ \Rightarrow \quad v_{\text{max}}^2 = \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} \]
\[ = \frac{\sin 15^\circ}{1.0 \cos 15^\circ} = 1.73 \]
\[ \Rightarrow \quad v_{\text{max}} = 34.4 \text{ m/s} \]
**GENERAL PRINCIPLES**

**Newton's Third Law**
Every force occurs as one member of an action/reaction pair of forces. The two members of an action/reaction pair:

- Act on two different objects.
- Are equal in magnitude but opposite in direction:
  \[ \overrightarrow{F}_{A \text{ on } B} = -\overrightarrow{F}_{B \text{ on } A} \]

**Solving Interacting-System Problems**

**MODEL** Choose the systems of interest.

**VISUALIZE**
  Pictorial representation:
  - Sketch and define coordinates.
  - Identify acceleration constants.
  Physical representation:
  - Draw a separate free-body diagram for each system.
  - Connect action/reaction pairs with dotted lines.

**SOLVE** Write Newton's second law for each system.
  Include all forces acting on each system.
  Use Newton's third law to equate the magnitudes of action/reaction pairs.
  Include acceleration constraints and friction.

**ASSESS** Is the result reasonable?
**IMPORTANT CONCEPTS**

Interacting systems and the environment:

Two systems interact by exerting forces on each other. Systems whose motion is not of interest form the environment. The systems of interest interact with the environment, but those interactions can be considered external forces.
APPLICATIONS

Acceleration constraints

Objects that are constrained to move together must have accelerations of equal magnitude: \( a_A = a_B \).

This must be expressed in terms of components, such as \( a_{Ax} = -a_{By} \).

Strings and pulleys

The tension in a string or rope pulls in both directions. The tension is constant in a string if the string is:

- Massless, or
- In equilibrium.

Systems connected by massless strings passing over massless, frictionless pulleys act as if they interact via an action/reaction pair of forces.
The 1.0 kg block in Figure is tied to the wall with a rope. It sits on top of a 2.0 kg block. The lower block is pulled to the right with a tension force of 20.0 N. The coefficient of kinetic friction at both lower and upper surfaces of 2 kg block is $\mu_k = 0.40$. (i) What is tension in rope holding 1.0 kg block? (ii) What is acceleration of 2.0 kg block?
\[ n = c + m_2 g \]
\[ T_2 - f_{k_1} - f_{k_2} = m_2 a \]
\[ c = m_1 g \quad T_1 = f_{k_1} \]
\[ f_{k_2} = \mu_k n \quad f_{k_1} = \mu_k c' = \mu_k m_1 g = 3.1 \mu_k \]
\[ T_2 = \mu_k m_1 g - \mu_k (m_1 g + m_2 g) = m_2 a \]
\[ 20 - 3.92 - 0.4 (3)(9.8) = 2a \]
\[ a = 2.16 \frac{m}{s^2} \]
What is the acceleration of 2.0 kg block in figure across the frictionless table?

Hint: Think carefully about the acceleration constraint.
\[ m_{1}g - T = m_{1}a_{1} \quad \rightarrow \quad 0 \]

\[ T = \frac{m_{2}a_{2}}{2} \]

\[ a_{2} = \frac{a_{1}}{2} \quad \Rightarrow \quad 27: \quad m_{2}a_{2} = \frac{m_{2}a_{1}}{2} \]

\[ T + T = m_{2}a_{2} \]

\[ T = \frac{m_{2}a_{1}}{4} \quad \rightarrow \quad 0 \]

\[ 0 + 0 = m_{1}g = (m_{1} + m_{2})a_{1} \]

\[ a_{1} = \left( \frac{m_{1}}{m_{1} + m_{2}} \right)g = \left( \frac{1}{1 + \frac{1}{2}} \right)g = \frac{2}{3} g \]

\[ a_{2} = \frac{a_{1}}{2} = \frac{1}{3} g = \frac{1}{3} (9.80) \text{ m/s}^{2} \]

\[ = 3.27 \text{ m/s}^{2} \]
**GENERAL PRINCIPLES**

**Law of Conservation of Momentum**

The total momentum \( \vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots \) of an isolated system is a constant. Thus

\[
\vec{P}_t = \vec{P}_i
\]

**Law of Conservation of Angular Momentum**

The angular momentum \( L \) of a particle or system of particles in circular motion does not change unless there is a net tangential force. Thus

\[
L_t = L_i
\]

**Solving Momentum Conservation Problems**

**Model** Choose an isolated system or a system that is isolated during at least part of the problem.

**Visualize** Draw a pictorial representation of the system before and after the interaction.

**Solve** Write the law of conservation of momentum in terms of vector components

\[
(p_{tx})_1 + (p_{tx})_2 + \cdots = (p_{tx})_1 + (p_{tx})_2 + \cdots
\]

\[
(p_{ty})_1 + (p_{ty})_2 + \cdots = (p_{ty})_1 + (p_{ty})_2 + \cdots
\]

**Assess** Is the result reasonable?
**IMPORTANT CONCEPTS**

**Momentum**  \( \vec{p} = m \vec{v} \)

**Impulse**  \( J_x = \int F_x(t) \, dt = \text{area under force curve} \)

Impulse and momentum are related by the impulse-momentum theorem:

\[ \Delta p_x = J_x \]

This is an alternative statement of Newton's second law.

Angular momentum  \( L = mrv \)

**System**  A group of interacting particles.

**Isolated system**  A system on which there are no external forces or the net external force is zero.

**Before-and-after pictorial representation**

- Define the system.
- Use two drawings to show the system before and after the interaction.
- List known information and identify what you are trying to find.

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APPLICATIONS

Collisions: Two or more particles come together.

In a perfectly inelastic collision, they stick together and move with a common final velocity.

Explosions: Two or more particles move away from each other.

Two dimensions: No new ideas, but both the x- and y-components of P must be conserved.

Momentum bar charts display the impulse-momentum theorem: \( P_f = P_i + \Delta P \).
A firecracker in a coconut blows coconut into 3 pieces. Two pieces of equal mass fly off south and west, perpendicular to each other at 20.0 m/s. The third piece has twice the mass as other two. What are speed and direction of the third piece?
\[ \Sigma P_x = \frac{M}{2} \cos \theta (V) - \frac{M}{4} (20 \text{ m/s}) = 0 \]

\[ \Sigma P_y = \frac{M}{2} \sin \theta (V) = \frac{M}{4} (20 \text{ m/s}) = 0 \]

\[ \Rightarrow \sin \theta = \frac{\sigma_0}{2} ; \quad \theta = 45^{\circ} \]

\[ V \cos \theta = 10 \text{ m/s} = 0 \]

\[ V = 14.1 \text{ m/s} ; \quad \text{NORTHEAST.} \]