Neutrino Mixing Pattern and its Implications for Charged Lepton Rare Decays and CP Violation

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Outline

Interpreting Neutrino Oscillation Results

- *Patterns of Neutrino Mass Spectrum*

Theoretical Modeling

- *Large Neutrino Mixing*
- *Unified Quark-Lepton Description*
- *Anomalous U(1) Symmetry and Fermion Mass Hierarchy*

Lepton Flavor Violation and Large Neutrino Mixings

- *Rare Decays* $\tau \to \mu\gamma$, $\mu \to e\gamma$
- *Leptogenesis*
- *Lepton Dipole Moments*

Conclusions

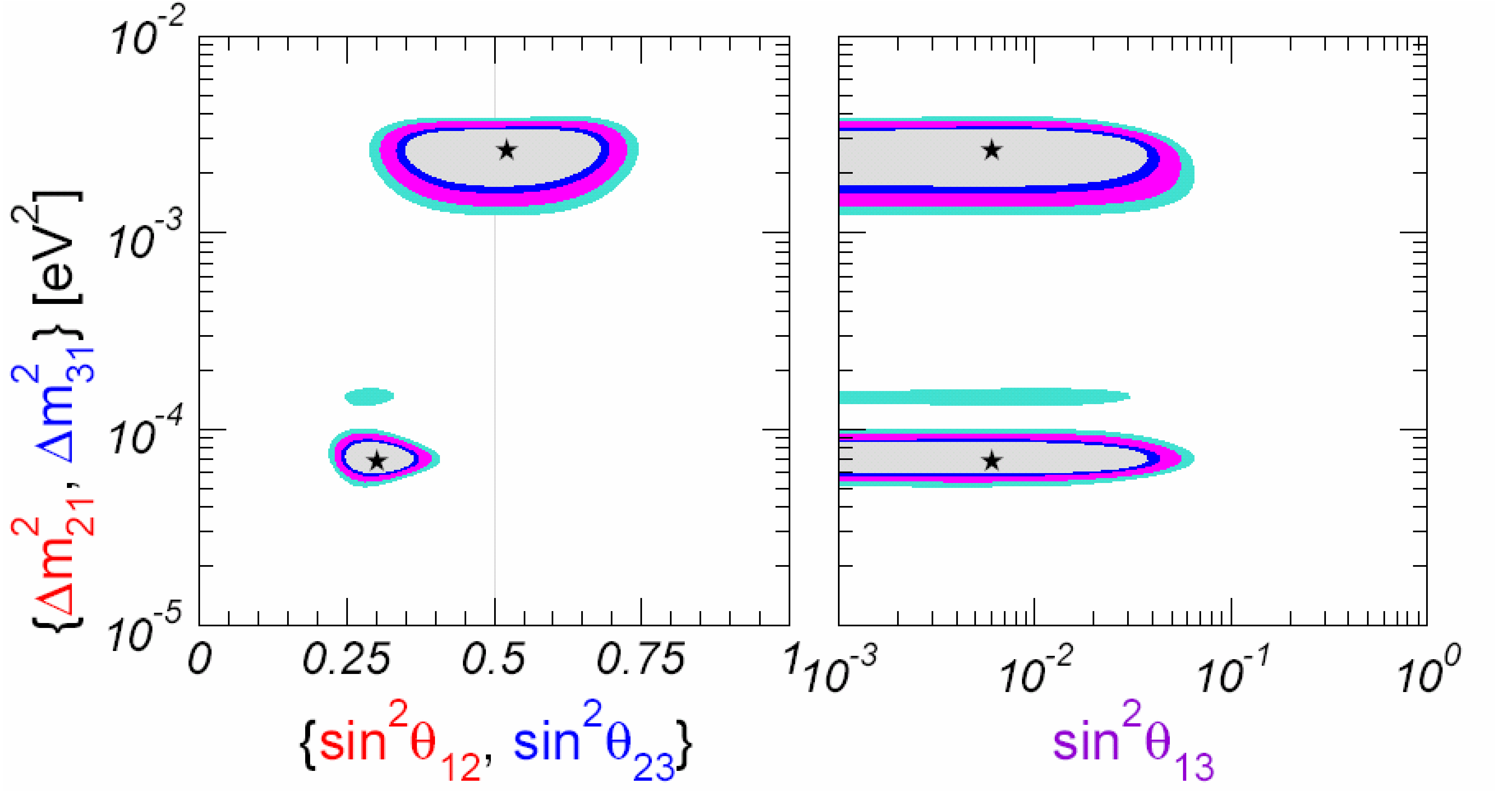
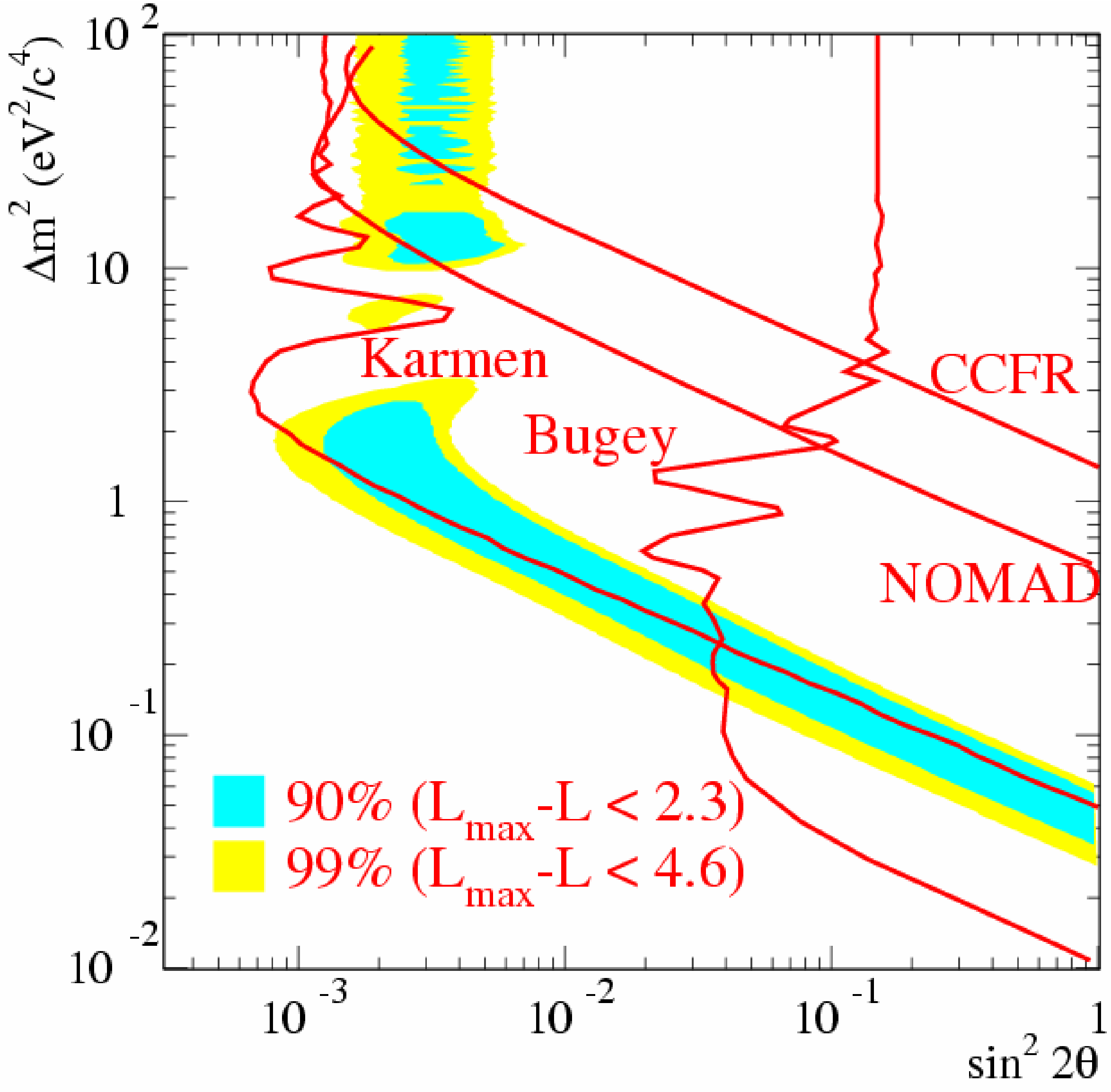


Figure 10: Projections of the allowed regions from the global oscillation data at 90%, 95%, 99%, and 3σ C.L. for 2 d.o.f. for various parameter combinations.

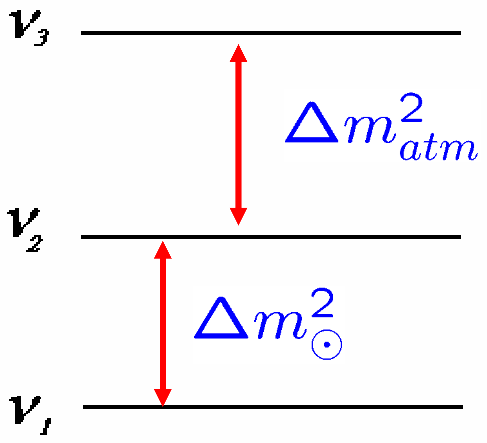
M. Maltoni et al., Phys. Rev. D68, 113010 (2003)

LSND

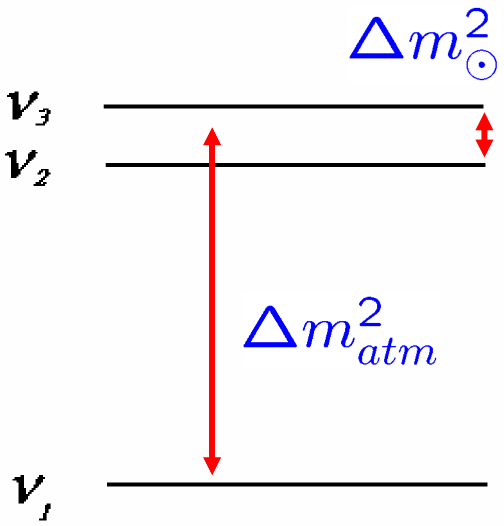


Aguilar, *et. al* hep-exp/0104049

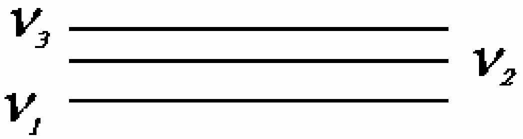
Patterns of Neutrino Mass Spectrum



Normal Hiearchy



Inverted Hiearchy



Quasi-degenerate

Neutrino Mixing versus Quark Mixing

Leptons

$$U_\ell = \begin{pmatrix} 0.85 & -0.52 & 0.053 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{pmatrix}$$

Quarks

$$V_q = \begin{pmatrix} 0.976 & 0.22 & 0.003 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{pmatrix}$$

Disparity a challenge for Quark-Lepton unified theories.

Neutrino Masses and the Scale of New Physics

$$\mathcal{L} = \frac{LLHH}{M_R}$$

$$\langle H \rangle \sim 246 \text{ GeV and } m_{\nu_3} \sim 0.05 \text{ eV}$$

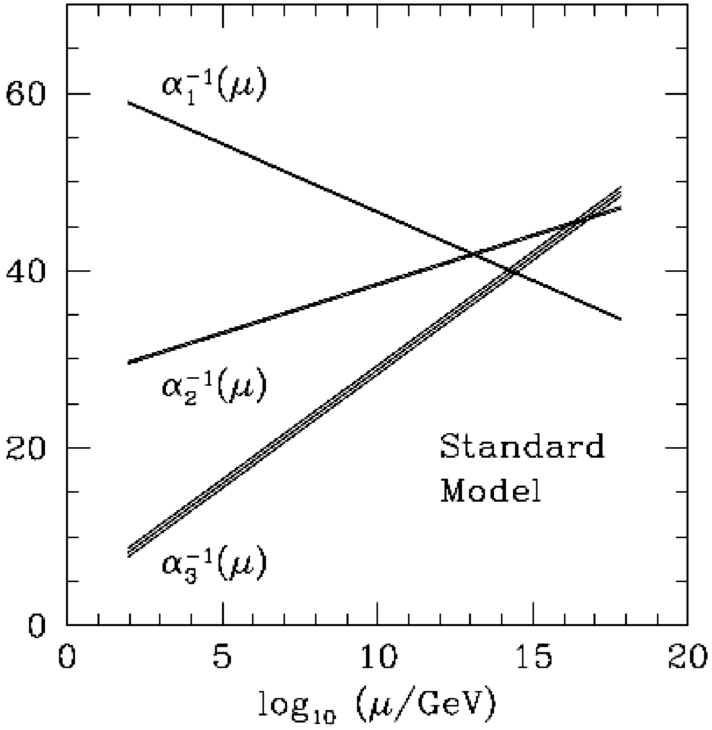
from atmospheric neutrino oscillation data

$$\Longrightarrow \qquad m_R \sim 10^{14} - 10^{15}\text{GeV}$$

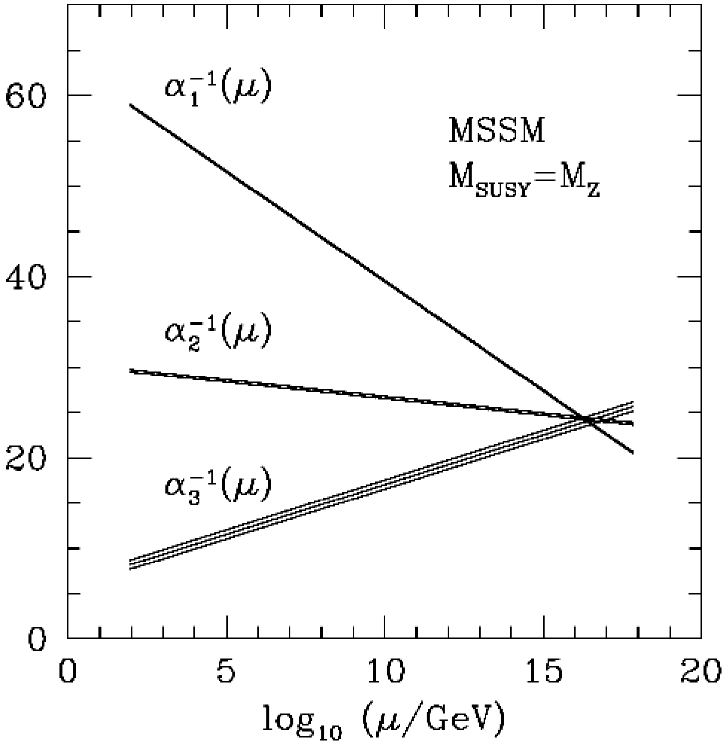
Very close to the GUT scale.

Leptogenesis via ν_R decay explains cosmological baryon asymmetry

Evolution of Gauge Couplings



Standard Model



Supersymmetry

Structure of Matter Multiplets

$$Q = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix} \sim (3, 2, \frac{1}{6})$$

$$u^c = \begin{pmatrix} u_1^c & u_2^c & u_3^c \end{pmatrix} \sim (\overline{3}, 1, \frac{-2}{3})$$

$$d^c = \begin{pmatrix} d_1^c & d_2^c & d_3^c \end{pmatrix} \sim (\overline{3}, 1, \frac{1}{3})$$

$$L = \begin{pmatrix} \nu \\ e^- \end{pmatrix} \sim (1, 2, \frac{-1}{2})$$

$$e^c \quad \sim (1, 1, +1)$$

$$\nu^c \quad \sim (1, 1, 0)$$

Matter Unification in 16 of SO(10)

$u_1 \; : \; |\uparrow\downarrow\uparrow\uparrow\downarrow>$

$u_2 \; : \; |\uparrow\downarrow\uparrow\downarrow\uparrow>$

$u_3 \; : \; |\uparrow\downarrow\downarrow\uparrow\uparrow>$

$d_1 \; : \; |\downarrow\uparrow\uparrow\uparrow\downarrow>$

$d_2 \; : \; |\downarrow\uparrow\uparrow\downarrow\uparrow>$

$d_3 \; : \; |\downarrow\uparrow\downarrow\uparrow\uparrow>$

$u_1^c \; : \; |\downarrow\downarrow\uparrow\downarrow\downarrow>$

$u_2^c \; : \; |\downarrow\downarrow\downarrow\uparrow\downarrow>$

$u_3^c \; : \; |\downarrow\downarrow\downarrow\downarrow\uparrow>$

$d_1^c \; : \; |\uparrow\uparrow\uparrow\downarrow\downarrow>$

$d_2^c \; : \; |\uparrow\uparrow\downarrow\uparrow\downarrow>$

$d_3^c \; : \; |\uparrow\uparrow\downarrow\downarrow\uparrow>$

$\nu \; : \; |\uparrow\downarrow\downarrow\downarrow\downarrow>$

$e \; : \; |\downarrow\uparrow\downarrow\downarrow\downarrow>$

$e^c \; : \; |\downarrow\downarrow\uparrow\uparrow\uparrow>$

$\nu^c \; : \; |\uparrow\uparrow\uparrow\uparrow\uparrow>$

SUSY Spectrum

	SM Particles	**SUSY Partners**	
Spin = 1/2	Q	$\tilde{Q}$	Spin = 0
	u^c	$\tilde{u}^c$	
	d^c	$\tilde{d}^c$	
	L	$\tilde{L}$	
	e^c	$\tilde{e}^c$	
Spin = 0	H_u	$\tilde{H}_u$	Spin = 1/2
	H_d	$\tilde{H}_d$	
Spin = 1	g	$\tilde{g}$	Spin = 1/2
	W	$\tilde{W}$	
	B	$\tilde{B}$	

$$R = (-1)^{3B+L+2S}$$

Lepton Flavor Violation and Neutrino Mass

Seesaw mechanism naturally explains small ν–mass.

$$\mathcal{L} = \bar{\nu}_L M_D \nu_R + \frac{1}{2} {\nu_R}^T M_R \nu_R + h.c.$$

$$M_\nu = -M_D M_R^{-1} M_D^T$$

Current neutrino-oscillation data suggests

$$M_R \sim (10^{12} - 10^{15})\ \mathrm{GeV}$$

Flavor change in neutrino-sector

$\Longrightarrow$ **Flavor change in charged leptons**

In standard model with seesaw, leptonic flavor changing is very tiny.

$$Br(\mu \to e\gamma) \propto \frac{1}{M_R^4} \sim 10^{-50}$$

In Supersymmetric Standard Model

$$Br(\mu \to e\gamma) \propto \frac{1}{M_{SUSY}^4} \sim 10^{-10}$$

For $M_R \leq \mu \leq M_{Pl}$ ν_R **active**

⇨ **flavor violation in neutrino sector transmitted to Sleptons**

Borzumati, Masiero (1986)

Hall, Kostelecky, Raby (1986)

Hisano et. al., (1995)

Hisano, Okada (1998)

SUSY Seesaw Mechanism

$$\mathcal{W} = f\nu^c\nu^c\Delta + Y_\nu \nu^c L H_u$$

$$M_D = Y_\nu v_u \; ; \; M_R = f v_{B-L}$$

If *B-L* is gauged, M_R must arise through Yukawa couplings.

Flavor violation may reside entirely in *f* or entirely in Y_ν

If flavor violation occurs only in Dirac Yukawa Y_ν (with mSUGRA)

$$\Delta m^2_{ij}(i \neq j) \simeq -\frac{1}{8\pi^2}(3m_0^2+A_0^2)(Y_\nu^\dagger Y_\nu)_{ij}\left(\ell n\frac{M_{Pl}}{M_{B-L}}\right)$$

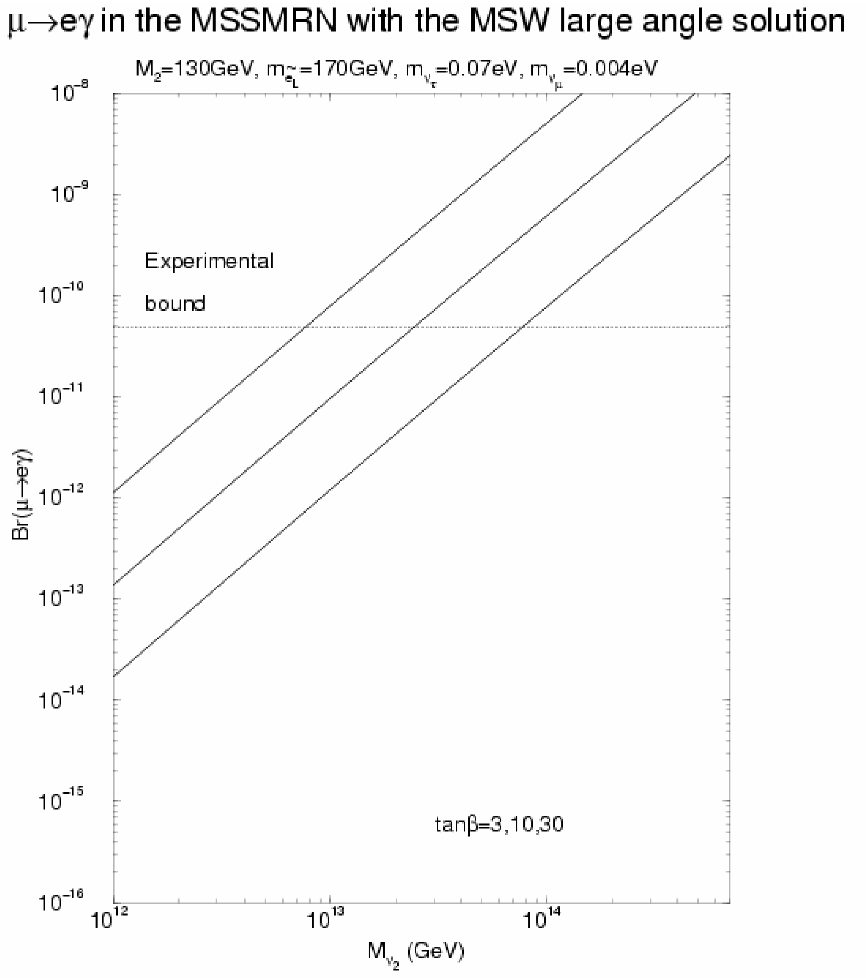
If flavor violation occurs only in *f* (Majorana LFV)

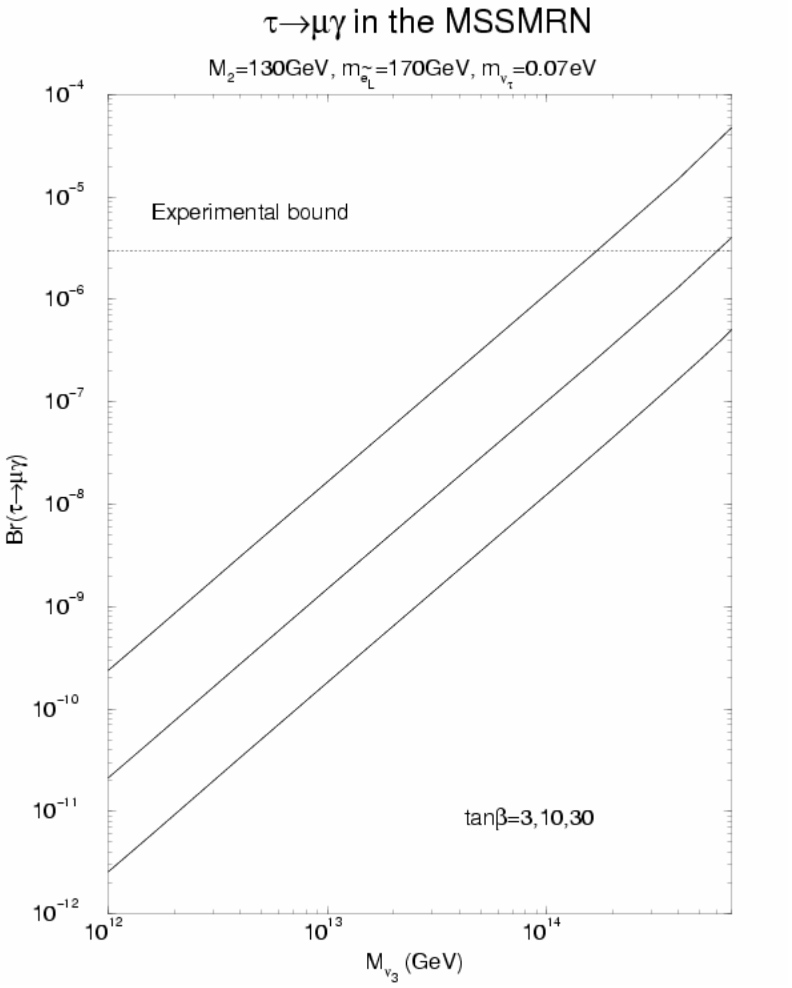
$$A_{\ell ij}(i \neq j) \;\simeq\; \frac{-3}{64\pi^4}[A_\ell(Y_\nu^\dagger Y_\nu f^\dagger f + f^\dagger f Y_\nu^\dagger Y_\nu)]_{ij}\left(\ell n\frac{M_{P\ell}}{M_{B-L}}\right)^2$$

$$\Delta m^2_{ij}(i \neq j) \;\simeq\; \frac{-3(m_0^2+A_0^2)}{32\pi^4}[Y_\nu^\dagger Y_\nu f^\dagger f + f^\dagger f Y_\nu^\dagger Y_\nu]_{ij}\left(\ell n\frac{M_{P\ell}}{M_{B-L}}\right)^2$$

LFV in the two scenarios are comparable.

More detailed study is needed.


$\mu \to e\gamma$ in the MSSMRN with the MSW large angle solution


$\tau \to \mu\gamma$ in the MSSMRN

LFV from Dirac neutrino Yukawa couplings

Hisano, Okada, (1998)

LFV from Majorana Yukawa Couplings

Minimal SUSY Left-Right Symmetric Model Dutta, Mohapatra, KSB (2002)

$$v_{B-L} = 2 \times 10^{12}\ \text{GeV},\ M_D \propto M_{l^+}$$

$$f = \begin{pmatrix} -1.1 \times 10^{-4} & -0.015 & 0.29 \\ -0.015 & 0.50 & -0.57 \\ 0.29 & -0.57 & 0.104 \end{pmatrix}$$

$$(m_1,\ m_2,\ m_3) = (-2.7 \times 10^{-3},\ 6.4 \times 10^{-3},\ 8.6 \times 10^{-2})\ \text{eV}$$

$$\mathbf{U} = \begin{pmatrix} 0.85 & -0.52 & -0.053 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{pmatrix}$$

Relevant for Leptogenesis: Baryon asymmetry can be related to neutrino oscillation parameters

For Dirac LFV scenario

$$M_R = (9 \times 10^{13}\ GeV) \times (\text{Identity Matrix})$$

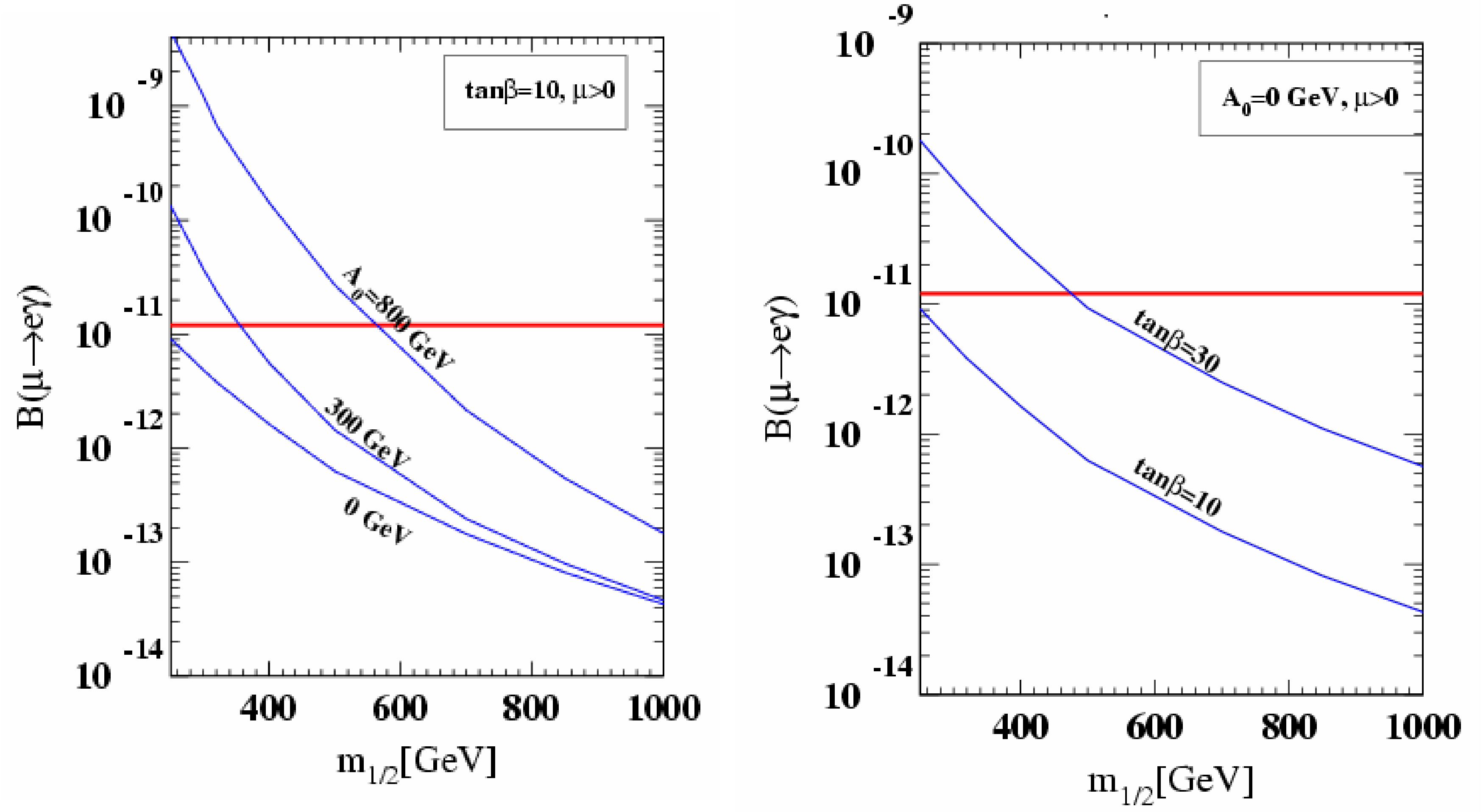
$$Y_\nu = \begin{pmatrix} 0.04 + 0.074i & -0.073 + 0.029i & 0.025 - 0.034i \\ -0.073 + 0.029i & -0.22 + 0.011i & -0.35 - 0.013i \\ 0.025 - 0.034i & -0.35 - 0.013i & -0.24 + 0.016i \end{pmatrix}$$

⇨ **Same neutrino oscillation parameters as in Majorona LFV**

The two scenarios differ in predictions for

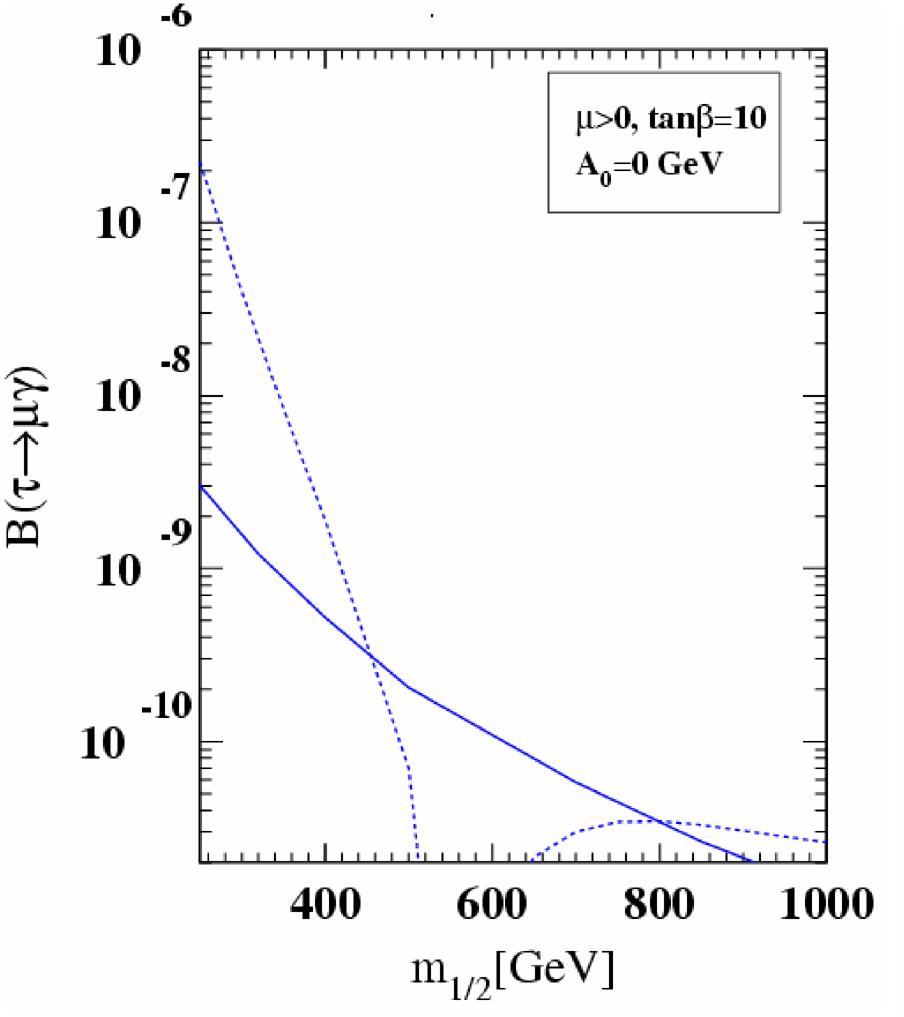
$$\begin{array}{ccc} \mu & \longrightarrow & e\gamma \\ \tau & \longrightarrow & \mu\gamma \\ \tau & \longrightarrow & e\gamma \end{array}$$

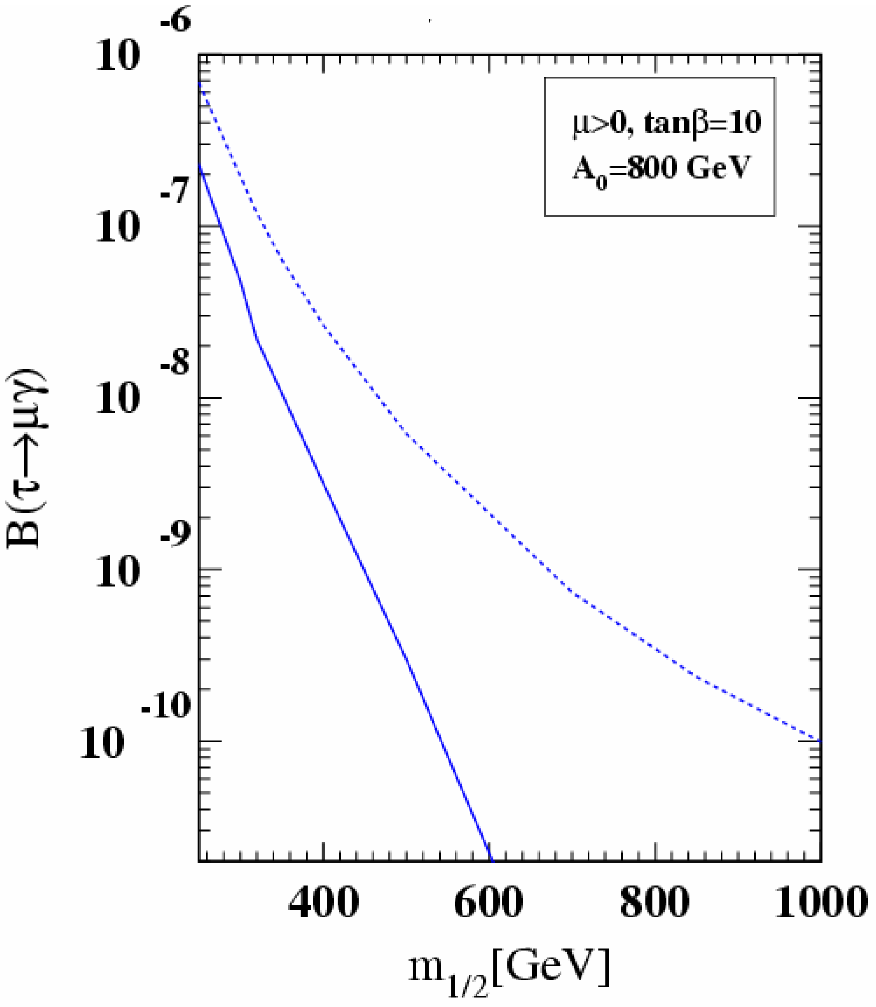
$\mu \to e\gamma$ **Majorana LFV**



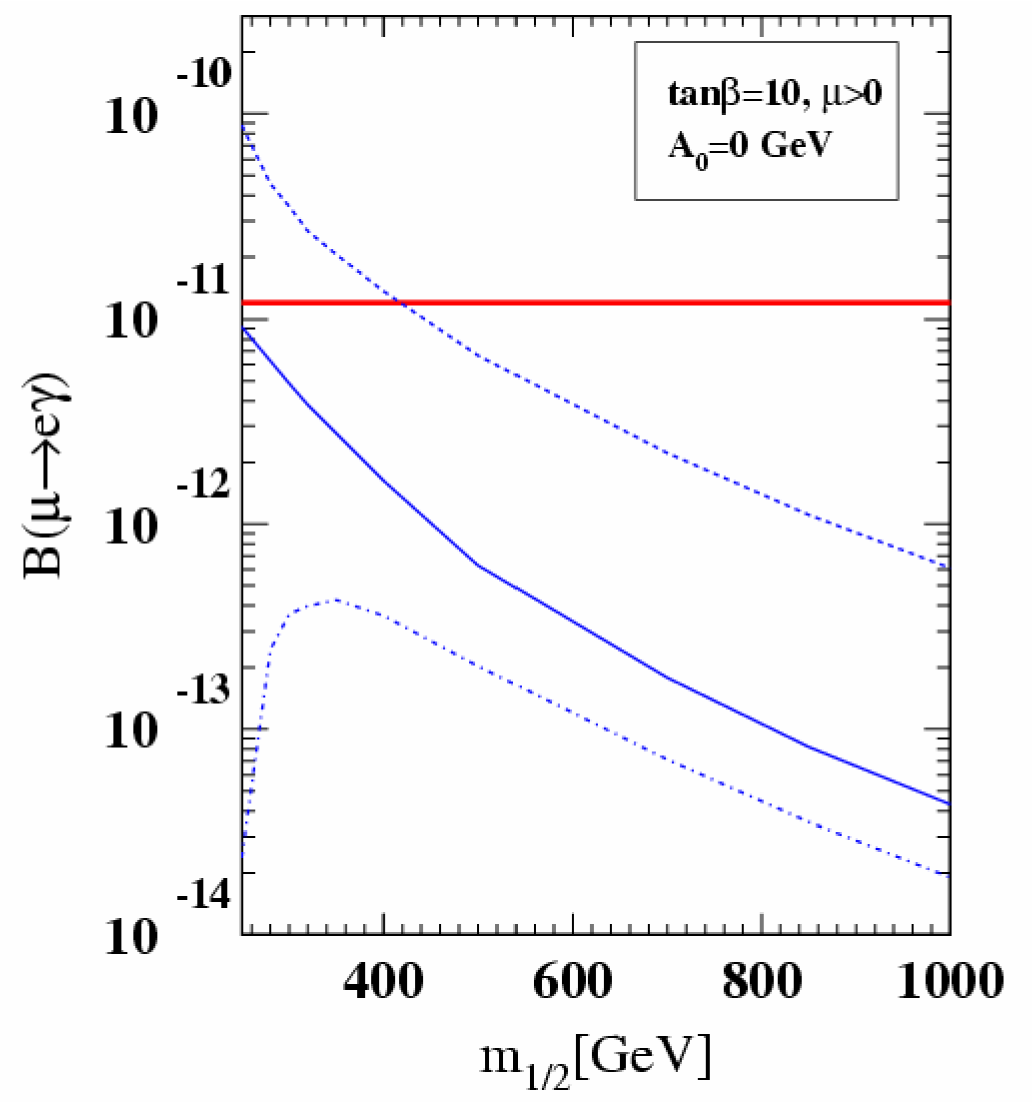
Dutta, Mohapatra, KSB (2002)

$\tau \to \mu\gamma$ Majorana LFV





Dirac versus Majorana LFV



Solid line: Majorana LFV

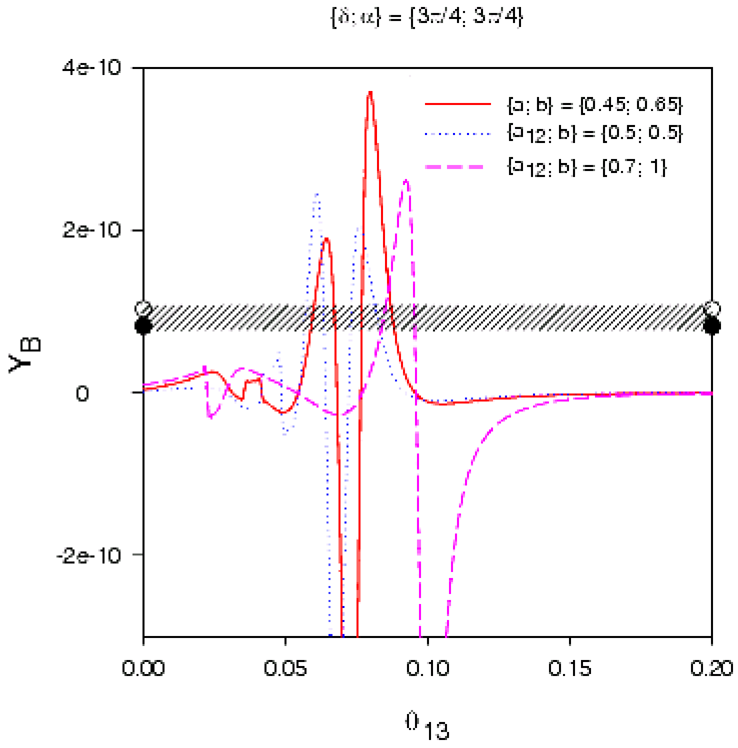
Dotted line: Dirac LFV

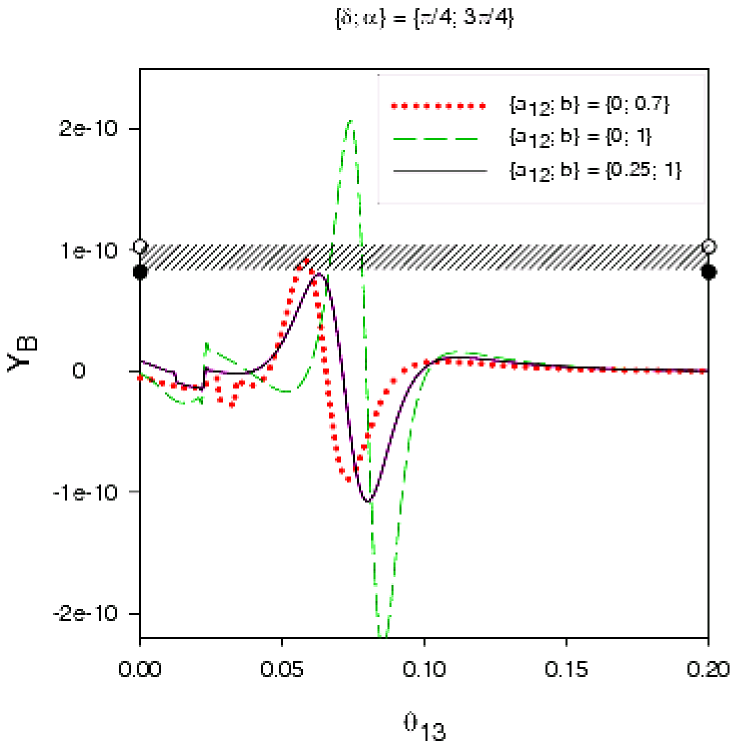
Dutta, Mohapatra, KSB (2002)

Baryon Asymmetry in the Minimal SUSY Left-Right Model

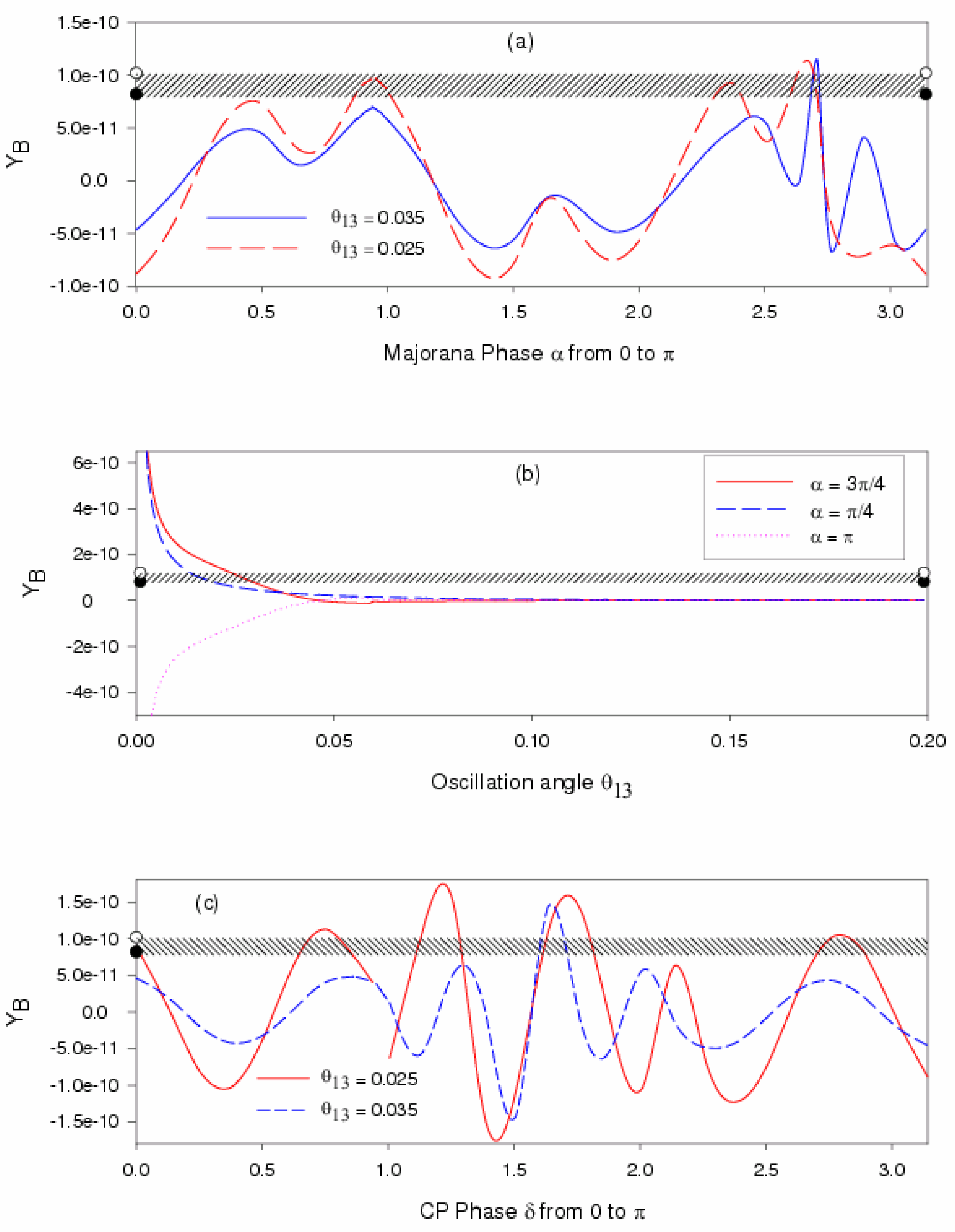
Seesaw sector has only 9 parameters (SM seesaw has 18 parameters)

Y_B determined completely in terms of neutrino mixing parameters





Bachri, KSB (2004)


(a)
Y_B
1.5e-10
1.0e-10
5.0e-11
0.0
-5.0e-11
-1.0e-10
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\theta_{13} = 0.035$
$\theta_{13} = 0.025$
Majorana Phase α from 0 to π
(b)
Y_B
6e-10
4e-10
2e-10
0
-2e-10
-4e-10
0.00
0.05
0.10
0.15
0.20
α = 3π/4
α = π/4
α = π
Oscillation angle θ_{13}
(c)
Y_B
1.5e-10
1.0e-10
5.0e-11
0.0
-5.0e-11
-1.0e-10
-1.5e-10
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\theta_{13} = 0.025$
$\theta_{13} = 0.035$
CP Phase δ from 0 to π

Large Neutrino Mixing with Lopsided Mass Matrices

Quark and Lepton Mass hierarchy:

$$m_d : m_s : m_b \sim m_e : m_\mu : m_\tau \sim \epsilon_1 : \epsilon_2 : \epsilon_3$$

$$m_u : m_c : m_t \sim \epsilon_1^2 : \epsilon_2^2 : \epsilon_3^2$$

This motivates:

$$\begin{aligned} U &= H^T U_0 H \\ D &= D_0 H \\ L &= H^T L_0 \\ N &= N_0 \end{aligned}$$

$$H = \mathrm{Diag}(\epsilon_1,\ \epsilon_2,\ \epsilon_3) \qquad \epsilon_1 << \epsilon_2 << \epsilon_3$$

10_i of $SU(5)$ carry flavor charge, $\bar{5}_i$ do not.
Leads to large left-handed charged lepton mixing and large right-handed down quark mixing.

S. Barr, KSB (1995)

Albright, KSB and Barr (1998)

Sato and Yanagida (1998)

Irges, Lavignac, Ramond (1998)

Altarelli, Feruglio (1998)

Example of Lopsided Mass Matrices

Gogoladze, Wang, KSB (2003)

$$U_{ij} = \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} H_u, \qquad D_{ij} = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^p H_d,$$

$$L_{ij} = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon \\ \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \epsilon^p H_d, \qquad \nu^D_{ij} = \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^{a_1} H_u$$

$\epsilon \sim 0.2$

$$\nu^M_{ij} \propto \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^{a_2} \qquad \sim \mathrm{M}^{\nu}_{\mathrm{light}}$$

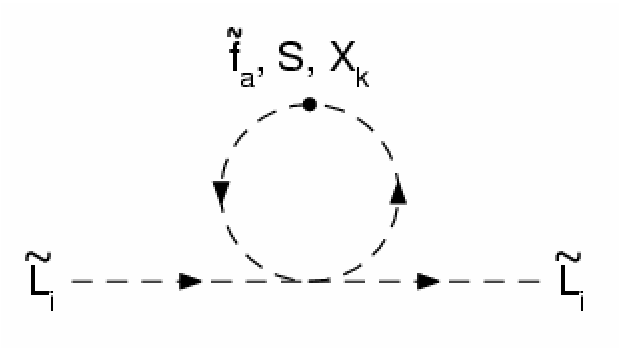
Discrete Z_N Gauge Symmetry

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	H_u	H_d	θ	S	A_2	A_3
A	0,2,6	1,3,7	3,5,5	4,6,6	13,1,5	5,7,7	1	13	7	2	6	13
B	4,6,10	13,1,5	11,13,13	6,8,8	9,11,1	5,7,7	13	1	7	2	13	13
C	6,8,12	5,7,11	1,3,3	0,2,2	7,9,13	5,7,7	9	5	7	2	13	6

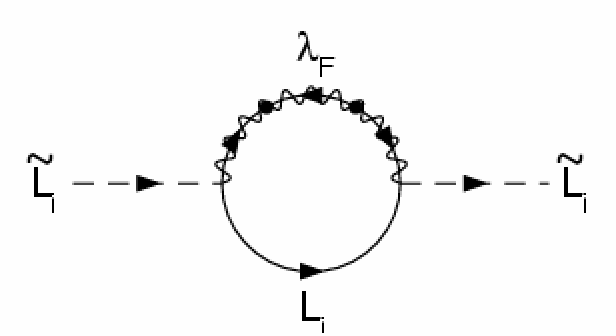
U(1) flavor symmetry will lead to observable charge lepton flavor violation.

Anomalous U(1) Symmetry and Lepton Flavor Violation

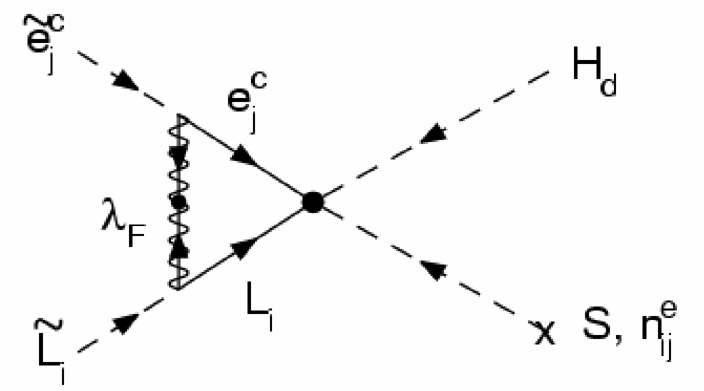
Enkhbat, Gogoladze, KSB (2003)



$$\delta\left(\tilde{m}_L^2\right)^A_{ij} \simeq -q_i^L|q_s|g_F^2\delta_{ij}m_0^2\mathrm{Tr}(Q)\frac{\ln\left(M_{st}/M_F\right)}{8\pi^2}$$

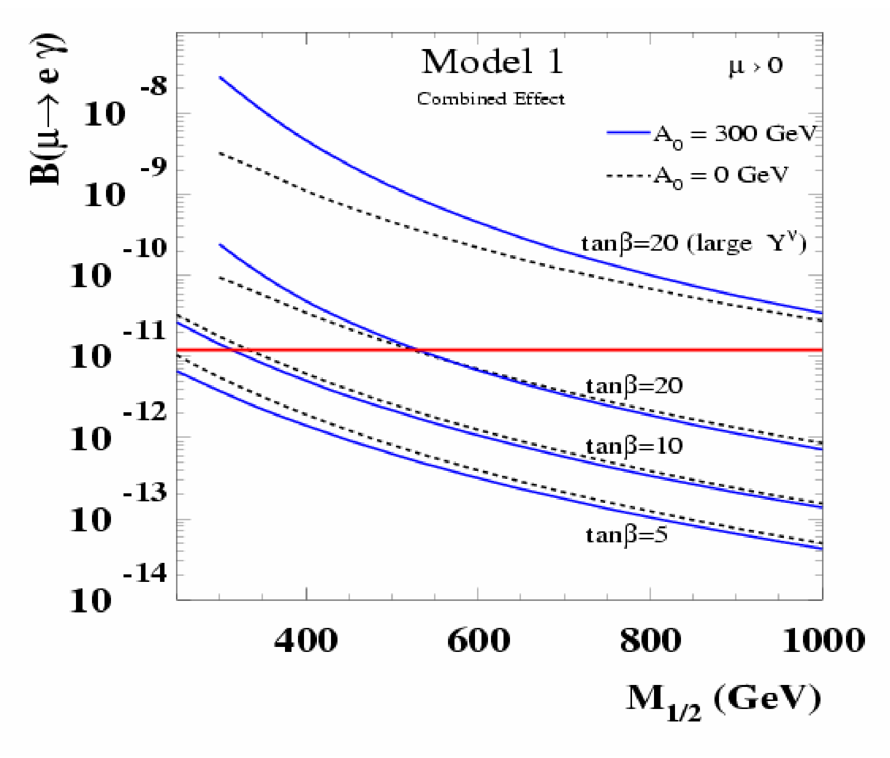


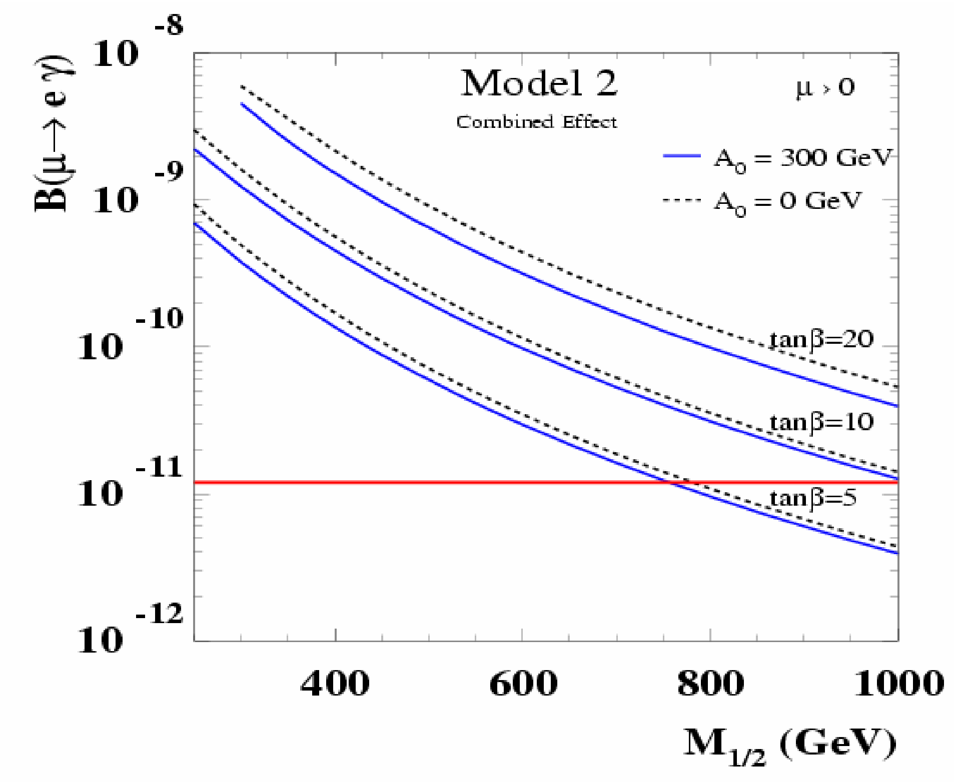
$$\delta\left(\tilde{m}_L^2\right)^G_{ij} \simeq \left(q_i^L\, g_F\right)^2\delta_{ij}\left(M_{\lambda_F}\right)^2\frac{\ln\left(M_{st}/M_F\right)}{2\pi^2}$$



$$\delta A^e_{ij} \simeq -M_{\lambda_F}g_F{}^2Y^e_{ij}Z^e_{ij}\frac{\ln\left(M_{st}/M_F\right)}{4\pi^2}$$

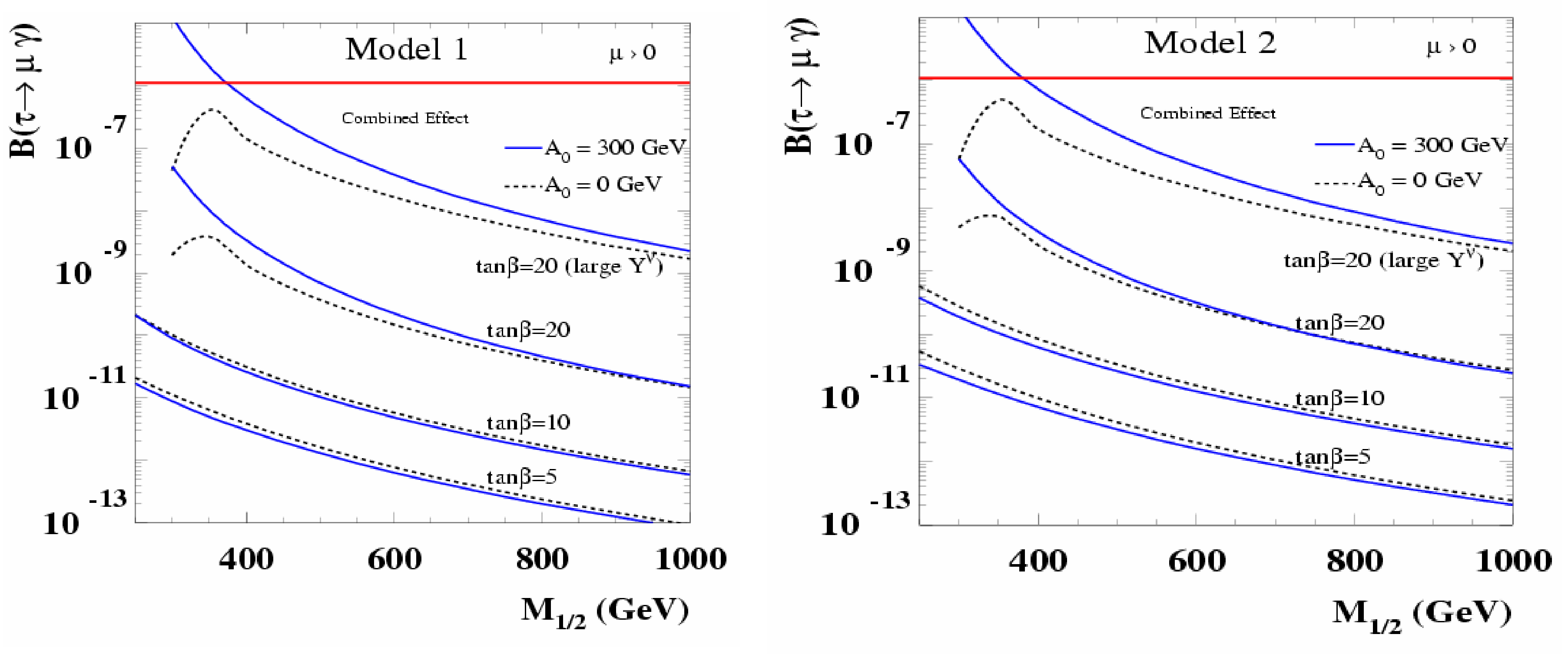
$\mu \to e\gamma$ in Anomalous $U(1)$ Models





Enkhbat, Gogoladze, KSB (2003)

$\tau \to \mu\gamma$ in Anomalous $U(1)$ Models



Enkhbat, Gogoladze, KSB (2003)

Electric Dipole Moments of the leptons and the neutron

$$\mathcal{L}_{eff} = -\frac{i}{2} d_f \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu}$$

Violates CP

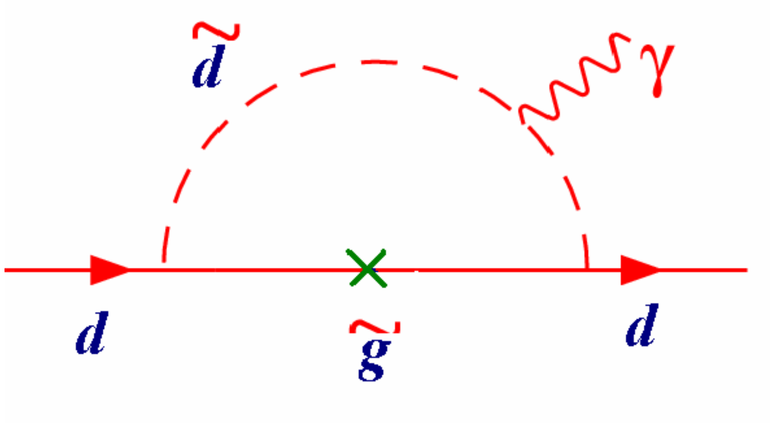
Electron: $d_e(\mathrm{Exp}) \leq 1.6 \times 10^{-27}$ e-cm

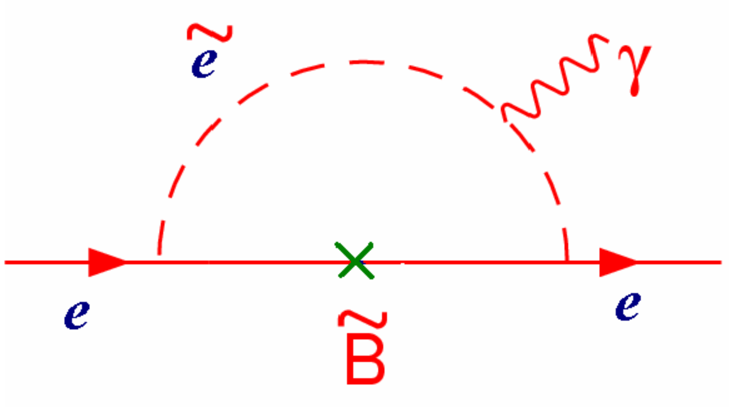
Muon: $d_\mu(\mathrm{Exp}) \leq 1.2 \times 10^{-18}$ e-cm

Neutron: $d_n(\mathrm{Exp}) \leq 6.3 \times 10^{-26}$ e-cm

Phases in SUSY breaking sector contribute to EDM.

SUSY Contributions:





A, B are complex in MSSM

$$d_n \sim (\sin\phi)\ 10^{-23}\ \text{e-cm}$$

$$d_e \sim (\sin\phi)\ 10^{-24}\ \text{e-cm}$$

$$\Rightarrow \quad \phi \simeq 10^{-2} - 10^{-1}$$

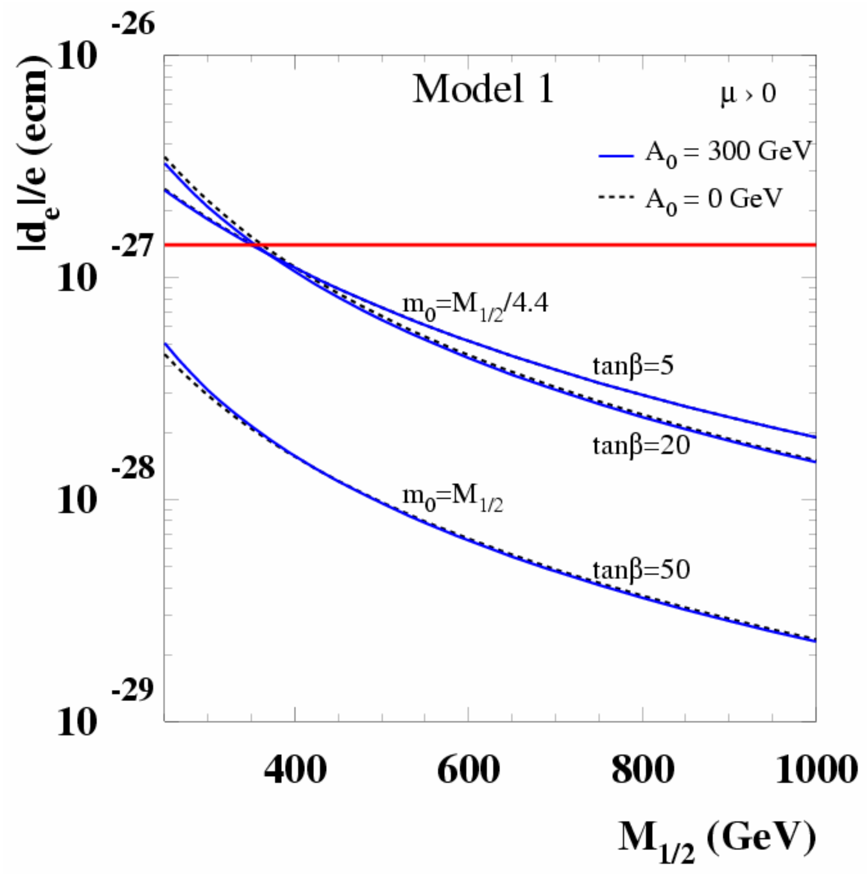
If SUSY-breaking parameters are all real, EDMs can still arise from neutrino Yukawa couplings

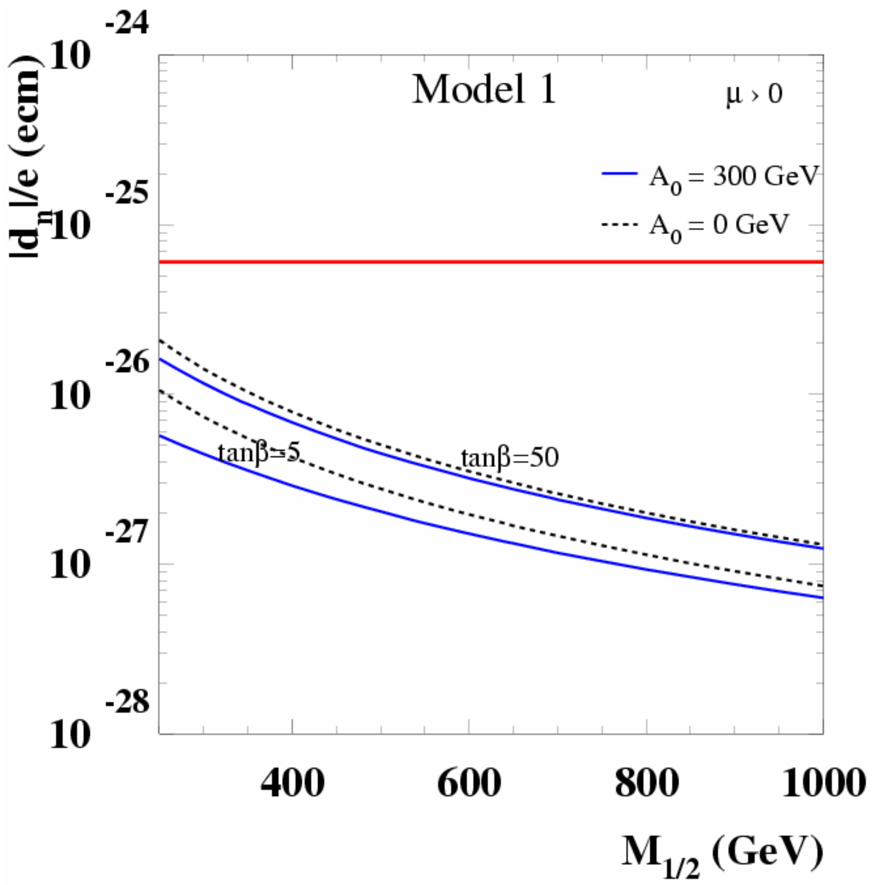
$d_e \sim 10^{-31}$ e-cm

Ellis et al (2002)

EDM of the electron and the neutron in Anomalous U(1) Models

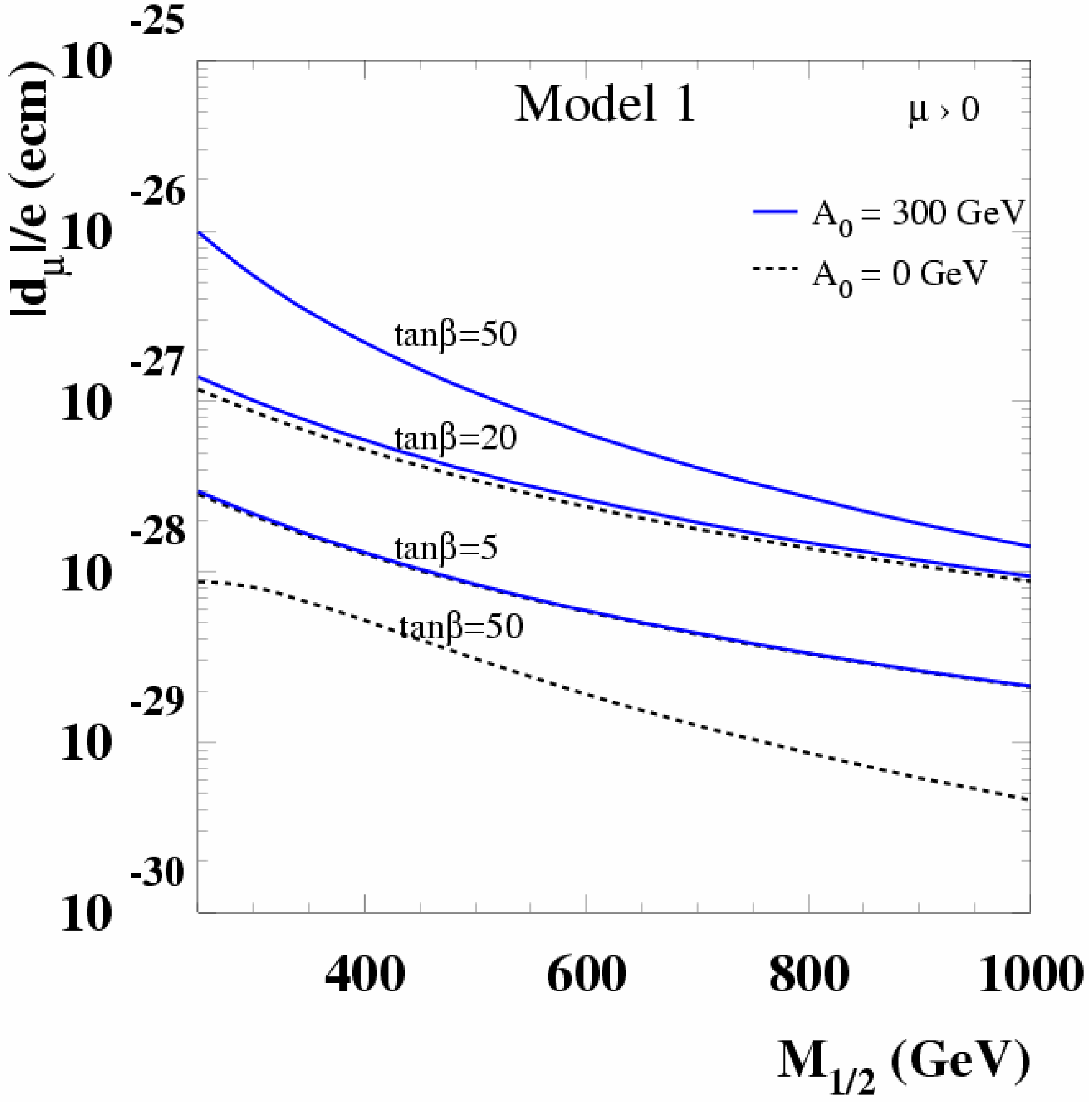
Complex Yukawa couplings, Real Soft SUSY breaking terms





Enkhbat, KSB (2004)

Muon EDM


Model 1
μ › 0
A$_0$ = 300 GeV
A$_0$ = 0 GeV
tanβ=50
tanβ=20
tanβ=5
tanβ=50
|d$_\mu$|/e (ecm)
10^{-25}
10^{-26}
10^{-27}
10^{-28}
10^{-29}
10^{-30}
400
600
800
1000
M$_{1/2}$ (GeV)

If parity is realized asymptotically,

$Y_U, \ Y_D, \ Y_E$ HERMITIAN

$A_U, \ A_D, \ A_E$ HERMITIAN

f Complex Symmetric

EDM will arise only through non-hermiticity induced by RGE.

$$d_e \simeq 10^{-28} - 10^{-27} \text{ e-cm};$$
$$d_n \simeq 10^{-26} - 10^{-27} \text{ e-cm}$$

Subject to experimental tests

$$d_\mu = 10^{-22} - 10^{-23} \text{ e-cm}$$

Dutta, Mohapatra, KSB (2001)

A$_4$ Symmetry and Quasi-degenerate Neutrino

E. Ma, 2002

E. Ma, J. Valle, KSB, 2002

$$L_i \sim 3, \qquad e_i^c : (1, 1', 1'')$$

$$\mathcal{M}_e = U_L \begin{bmatrix} h_1^{e\prime} & 0 & 0 \\ 0 & h_2^{e\prime} & 0 \\ 0 & 0 & h_3^{e\prime} \end{bmatrix} \frac{\sqrt{3} f_e v_1 u}{M_E}, \quad U_L = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

$$\mathcal{M}_\nu = \frac{f_N^2 v_2^2}{M_N} U_L^T U_L = \frac{f_N^2 v_2^2}{M_N} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

With Arbitrary Soft A$_4$ Breaking

$$\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta/\sqrt{2} & \sin\theta/\sqrt{2} \\ -\sin\theta & \cos\theta/\sqrt{2} & \cos\theta/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}$$

With Complex parameters, $\arg(U_{e3}) = \pi/2$

Inverted Hierarchy Models

$$M_\nu = \begin{pmatrix} 0 & A & B \\ A & 0 & 0 \\ B & 0 & 0 \end{pmatrix}$$

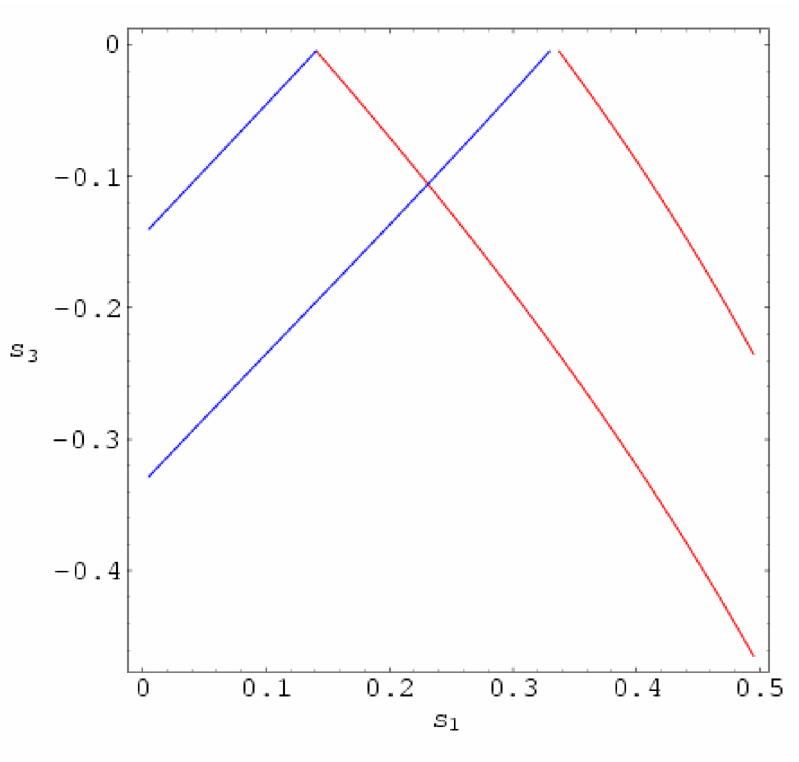
$L_e - L_\mu - L_\tau$ Symmetry

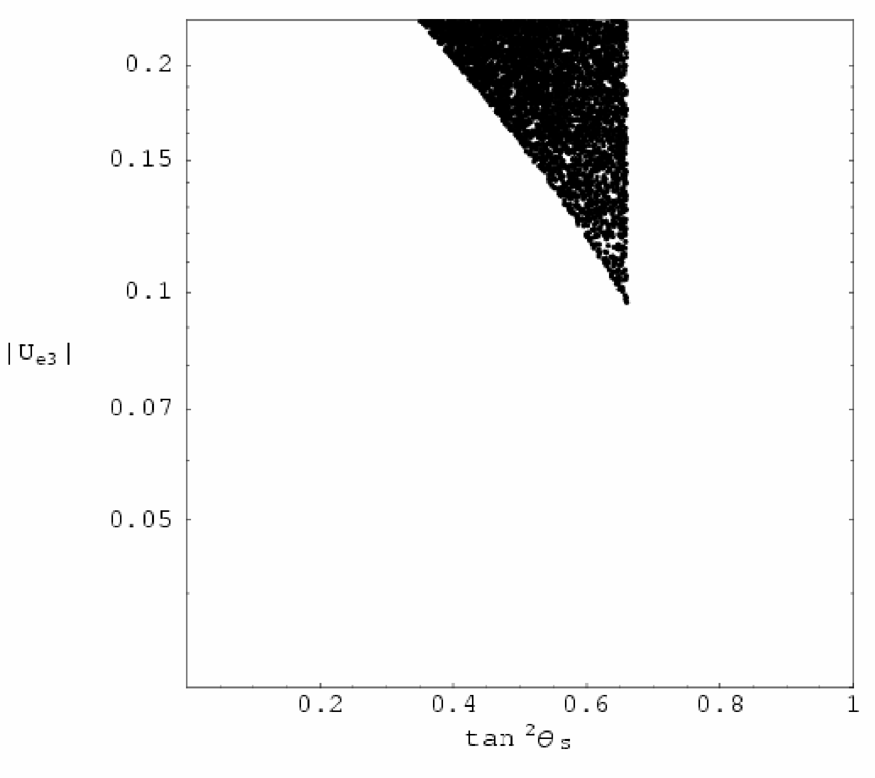
⇒ Bimaximal Neutrino Mixing

Symmetry breaking has to be strong in order to accommodate the Solar oscillation angle

Symmetry breaking needed in the charged lepton mass matrix

L_e- L_μ- L_τ Models





Joshipura, KSB (2004)

Conclusions

- Neutrino experiments pinning down fundamental parameters of theory
- Large neutrino mixings can arise from unified theories through lopsided mass matrices
- Anomalous U(1) models natural candidates for explaining fermion masses and neutrino mixings
- Lepton Flavor Violation $\tau \to \mu\gamma$, $\mu \to e\gamma$, and EDMs within reach of experiments