Link Between Baryon Asymmetry and Neutrino Oscillation Parameters
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OUTLINE

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• Fermion Mass Model
• Low Energy Constraints on CP Violation at High Energy
• Numerical Result $CP; Y_L$ and $Y_B$
• Conclusion
Introduction:

Experiments + Observation

\[ \Downarrow \]

Neutrino Oscillation

\[ \Downarrow \]

Neutrino Masses + Mixing

\[ \Downarrow \]

Extend The Standard Model by adding one r.h.n field per family

\[ \Downarrow \]

SeeSaw + Lepton Asymmetry
Lepton and Baryon Asymmetries:

\[-\mathcal{L} = \bar{l}_L (\phi Y_i) e_R + \bar{l}_L (\phi Y_i) \nu_R + \frac{1}{2} \bar{\nu}_R C M_R \bar{\nu}_R\]

\(N_i\) interact only weakly with other species and fall out of equilibrium at some \(T \sim M_R \gg T_{EW} \).

The interference between the 3-level and one loop decay amplitude give rise to CP violating asymmetry \(\varepsilon_1\):

\[\varepsilon_1 = \frac{\Gamma(\nu_R \to l\phi) - \Gamma(\nu_R \to \bar{l}\phi^*)}{\Gamma(\nu_R \to l\phi) + \Gamma(\nu_R \to \bar{l}\phi^*)}\]

These decays generate a non-zero lepton number.

(B-L) conserving SM sphaleron reactions which occurs rapidly enough above \(T_{EW}\) convert the lepton asymmetry into baryon asymmetry.
Questions:
1- Can CP asymmetry produced in the decay of r.h.n be related to Neutrino Oscillation parameters?
2- Can we explain the observed

\[ Y_B = \frac{n_B}{n_\gamma} \approx 6^{1.1}_{-0.5} \times 10^{-10} \]

Motivation:
In general, Lepton Flavor Mixing follows from a mismatch between the diagonalization of \( M_l \) and \( M_\nu \).

On the other hand, the only possible link between the high and low physics is through SeeSaw

\[ M_\nu = M_\nu^D M_R^{-1} M_\nu^D \]

So far, we only know the eigenvalue of \( M_\nu \) but we have no information on its eigenvectors. In some class of models

\[ M_\nu^D = \gamma \tan(\beta) \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \]

\( \Rightarrow \) All the low physics information will come from \( M_R \).
**Model:**
The texture of a fermion mass matrix has to be carefully chosen to guarantee acceptable agreement between the model predictions and the observational data.

\[ M_\nu^\nu = \gamma \tan(\beta) diag(m_e, m_\mu, m_\tau) = \gamma \tan(\beta) M_l \]

Babu, Mohapatra and Dutta.

**Implementing SeeSaw Mechanism**

\[ M_\nu = M_\nu^\nu M_R^{-1} M_\nu^T \]

solving for

\[ M_R = \gamma^2 \tan^2(\beta) M_l M_\nu^{-1} M_l \]

Consider the following light neutrino mass matrix:

\[ M_\nu = m \begin{pmatrix} ee^2 & h\epsilon & de \\ h\epsilon & 1 + a\epsilon & 1 \\ de & 1 & 1 + b\epsilon \end{pmatrix} \]

Where a, b, d, e and h are complex parameters of order 1. Then;

\[ M_R = \eta \begin{pmatrix} m_e^2(a + b + ab\epsilon) & m_em_\mu(d - h - h\epsilon) & m_e m_\tau(h - d - ade) \\ m_em_\mu(d - h - h\epsilon) & m_\mu^2(e\epsilon + e\epsilon^2 - ed^2) & m_\mu m_\tau(dh - e)\epsilon \\ m_em_\tau(h - d - ade) & m_\mu m_\tau(dh - e)\epsilon & m_\tau^2(e - h^2 + aee)\epsilon \end{pmatrix} \]

where \( \eta = \gamma^2 \tan^2(\theta)/m_\epsilon(2dh - d^2 - h^2) \)

Introduce the following parameters;

\[ a = \alpha \exp(i\xi), b = \beta \exp(i\rho), d = \delta \exp(i\Omega), e = \kappa \exp(i\chi) \text{ and } h = \sigma \exp(i\Lambda) \]

We also parameterize the masses as follows:

\[ m_\tau = m_3, m_e = a_1 \epsilon^3 m_3 \text{ and } m_\mu = a_2 \epsilon m_3 \]
$M_R$ is diagonalized by a unitary transformation $U$

$$U^T M_R U = M_R^{\text{diag}} \implies U^T M_R U^\dagger M_R^\dagger U^* = (M_R^{\text{diag}})^2$$

and

$$U^T (M_R M_R^\dagger) U^* = U^T M U^* = (M_R^{\text{diag}})^2 = \begin{pmatrix}
|M_1|^2 & 0 & 0 \\
0 & |M_2|^2 & 0 \\
0 & 0 & |M_3|^2
\end{pmatrix}$$

then we find

$$|M_1| = \frac{a_1^2 m_3^2 \gamma^2 \tan^2(\beta) \epsilon^4}{m \kappa}$$

$$|M_2| = \frac{a_2^2 m_3^2 \gamma^2 \kappa \tan^2(\beta) \epsilon^2}{m [\kappa^2 + \sigma^4 - 2 \kappa \sigma^2 \cos(2 \Lambda - \chi)]^{1/2}}$$

$$|M_3| = \frac{m_3^2 \gamma^2 \tan^2(\beta) [\kappa^2 + \sigma^4 - 2 \kappa \sigma^2 \cos(2 \Lambda - \chi)]^{1/2}}{m (\delta^2 + \sigma^2 - 2 \delta \sigma \cos(\Lambda - \Omega))}$$

We choose to evaluate the Dirac Neutrino mass matrix in the same basis where $M_R$ is diagonal.

$$\nu_R \longrightarrow U N_R$$

$$\overline{\nu_L} M'_D \nu_R = \overline{\nu_L} (M'_D U) \nu_R$$

$$M'_D = M'_D U$$

$$= \gamma \tan(\beta) m_3 \begin{pmatrix}
\omega \epsilon^3 & 0 & 0 \\
0 & \epsilon & 0 \\
0 & 0 & 1
\end{pmatrix} U$$

and call $M_D = M'_D U$

We now wish to evaluate the CP asymmetry $\varepsilon_1$. When $M_1 \ll M_2 \ll M_3$,

$$\varepsilon_1 = \frac{-3 M_1}{M_2} \frac{\text{Im}(M_D^\dagger M_D)^2_{21}}{8 \pi v^2 (M_D^\dagger M_D)_{11}}$$
**Low Energy Constraints:**

We are working in a basis where the charged lepton mass matrix is diagonal. The mixing information will be entirely contained in the neutrino mass matrix. The flavor basis and the mass basis are related by

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix}
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
\]

The form of \(U\) is given in general by

\[
U = U_{23}U_{13}U_{12}
\]

\(U_{ij}\) = rotation matrix about the \(i\)th and \(j\)th mass eigenstate.

For

\[
M_\nu = m \begin{pmatrix}
\kappa e^{i\chi} & \sigma e^{i\Lambda} & \delta e^{i\Omega} \\
\sigma e^{i\Lambda} & 1 + \alpha e^{i\xi} & 1 \\
\delta e^{i\Omega} & 1 & 1 + \beta e^{i\rho}
\end{pmatrix}
\]

we find

\[
m_{\nu_e} = m\sigma\epsilon\tan(\theta_{12})
\]

\[
m_{\nu_\mu} = m\sigma\epsilon\tan(\theta_{12})
\]

\[
m_{\nu_\tau} = 2m(1 + \frac{\epsilon}{4}(\alpha \cos(\rho) + \beta \cos(\rho)))
\]

and get

\[
\sigma = \frac{2\sqrt{\Delta m^2_\odot}}{\epsilon\sqrt{\Delta m^2_{atm}}} \cdot \frac{\tan\theta_{12}}{\sqrt{1 - \tan^4(\theta_{12})}}
\]

\[
\kappa = \frac{2m\beta\beta}{\epsilon^2\sqrt{\Delta m^2_{atm}}}
\]

\[
\delta = -\frac{\phi}{\epsilon}
\]
Establishing The Connection:

\[ \varepsilon_1 = f(\alpha, \beta, \delta, \sigma, \kappa, \Omega, \Lambda, \chi, \ldots) \epsilon^4 \]

\[ = \left( \frac{-3M_1}{M_2} \right) \text{Im}(M_D^\dagger M_D)_{21}^2 \]

\[ \frac{8\pi v^2 (M_D^\dagger M_D)_{11}}{ \epsilon^4} \]

where we find that:

\[ (M_1/M_2) = \frac{\epsilon^2 B_2^1}{\kappa} \]

\[ \text{Im}(M_D^\dagger M_D)_{21}^2 = \left[ \frac{(\gamma \tan(\beta)m_3)^2}{2} F\{ \frac{1}{C} + \frac{D}{B.A^2} \} \right]^2 \]

\[ (M_D^\dagger M_D)_{11} = (\gamma \tan(\beta)m_3)^2 \epsilon^4 \left[ \frac{E^2}{B.A^3} + \frac{A}{4.C} \right] \]

and

\[ A = \delta^2 + \sigma^2 - 2\delta\sigma \cos(\Lambda - \Omega) \]

\[ B = \kappa^2 + \sigma^4 - 2\kappa\sigma \cos(2\Lambda - \chi) \]

\[ C = \kappa^2 + \delta^2\sigma^2 - 2\delta\kappa\sigma \cos(\Lambda - \chi + \Omega) \]

\[ D = 3\delta^4 + 2\delta\sigma + 10\delta^2\sigma^2 + 3\sigma^4 - 12\delta\sigma(\delta^2 + \sigma^2) \cos(\Lambda - \Omega) + 12\delta(1 + 2\delta)\sigma^2 \cos(2(\Lambda - \Omega)) \]

\[ E = 3\delta^4 + 2\delta(1 + 5\delta)\sigma^2 + 3\sigma^4 + 12\delta(\delta^2 + \sigma^2) \cos(\Lambda - \Omega) + \delta(1 + 2\delta)\sigma \cos(2\Lambda - \Omega) \]

\[ F = (\delta\sigma^2 - \kappa\sigma)\sin(\Omega) \]

\[ G = \{BA^2 + DC\}^2 \]

\[ K = B^{\frac{1}{2}}\{4CE^2 + BA^4\}AC \]

\[ \varepsilon_1 = \frac{-3(\gamma \tan(\beta)m_3)^2 \ G.F^2}{8\pi v^2} \frac{1}{\kappa K} \epsilon^4 \]

and show that

\[ \frac{G.F^2}{\kappa K} \times \left\{ \frac{9.10^5 \sqrt{\Delta m_{atm}^2 \Delta m_{\odot}^2 (m_{\beta\beta} - 4 \sqrt{\Delta m_{atm}^2 \varphi^2})^2 \tan^2(\theta_{12})}}{m_{\beta\beta} \epsilon^4 \varphi^2 (\epsilon - 5\varphi + (\epsilon - 2\varphi) \cos(2\Omega))^2 (1 - \tan^4(\theta_{12}))^2} \right\} \sin^2(\Omega) \]

\[ \times \left\{ \frac{(m_{\beta\beta} \sqrt{\Delta m_{atm}^2 - 2\Delta m_{\odot}^2 \tan^2(\theta_{12}) - m_{\beta\beta} \sqrt{\Delta m_{atm}^2 \tan^4(\theta_{12})}})}{m_{\beta\beta} \epsilon^4 \varphi^2 (\epsilon - 5\varphi + (\epsilon - 2\varphi) \cos(2\Omega))^2 (1 - \tan^4(\theta_{12}))^2} \right\} \]
Numerical Result:

\[ Y_L \equiv \frac{n_L - n_{\bar{L}}}{s} = \frac{d \varepsilon_1}{g^*} \]

\[ d = \frac{0.3}{k (\ln(k))^{0.6}} \]

\[ k = \frac{M_{pl} (M_D^\dagger M_D)_{11}}{1.66 \sqrt{g^*} (8\pi v^2) M_1} \]

\[ Y_B = \frac{C}{C - 1} Y_L \]

where \( C = \frac{8N_f + 4N_\nu}{22N_f + 13N_\nu} \approx \frac{1}{3} \) for \( N_f = 3 \) and \( N_\nu = 1 \).

Observational data:
\( \Delta m_\odot^2 \approx 7.110^{-5} \); \( \Delta m_{atm}^2 \approx 2.710^{-3} \); \( m_{\beta\beta} \approx 10^{-4} \); \( \varphi_{13} \approx 0.16 \); \( \tan(\theta_{12}) \approx 0.58 \)

<table>
<thead>
<tr>
<th>( \Omega = 0.9 )</th>
<th>( \varepsilon_1 )</th>
<th>( k )</th>
<th>( d )</th>
<th>( Y_L )</th>
<th>( Y_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.35 ( 10^{-6} )</td>
<td>25.48</td>
<td>5.8 ( 10^{-3} )</td>
<td>-2.53 ( 10^{-10} )</td>
<td>8.43 ( 10^{-11} )</td>
<td></td>
</tr>
</tbody>
</table>

In which case the light neutrino masses are:

\[ \{ m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau} \} = \{ 0.00163789, 0.00497758, 0.0519079 \} \]

and the resulted PMNS:

\[
PMNS = \begin{pmatrix}
0.8212 & 0.5585 & 0.1167 \\
0.3454 & 0.6145 & 0.7092 \\
0.4541 & 0.5571 & 0.7005
\end{pmatrix}
\]
Conclusion:

The model is very predictive and allowed for reasonable obtained values for CP Asymmetry, Lepton Asymmetry and Baryon Asymmetry consistent with recent data.

The CP phase $\Omega$ has to be large and around 0.9 to produce enough CP asymmetry for leptogenesis.

We could directly link CP asymmetry in the r.h.n sector to low energy physics parameters and found it to be no less that $\sim 10^{-6}$ which again would be enough to produce the observable lepton and baryon asymmetry.

The link between the baryon asymmetry and low energy parameter is implicit through CP.